Problem Set 3

Problem 3 – Given the Lagrangian:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

and the Euler-Lagrange equation:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_i} \right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$$

the equation of motion for ψ is determined by substituting $\bar{\psi}$ into the Euler-Lagrange equation for ϕ_i. The first term is zero and the second term gives:

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0.$$

Likewise the equation of motion for $\bar{\psi}$ is

$$i\partial_\mu \bar{\psi}\gamma^\mu + m\psi = 0.$$

Under the transform

$$\psi \longrightarrow e^{i\theta}\psi$$
$$\bar{\psi} \longrightarrow e^{-i\theta}\bar{\psi}$$

where θ is independent, the Lagrangian is invariant. There is a $\bar{\psi}$ and a ψ in each term and the exponential phases cancel out. The current associated with this transformation is:

$$i\bar{\psi}\gamma_\mu \psi.$$

It is the current with couples to the E&M field as can be seen in the next problem.

Problem 4 – If θ is position dependent, then there is an additional term in the transformed Lagrangian of

$$-\bar{\psi}\gamma^{\mu}\partial_{\mu}\theta\psi.$$

To cancel this term out, you have to add a term to the Lagrangian of the form:

$$i\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

where $A_{\mu} \rightarrow A_{\mu} - i\partial_{\mu}\theta$. This is the E&M vector field.