Problem Set 2

Problem 4

$$\psi(t) = \psi(0)e^{-i\left(m-i\frac{\Gamma}{2}\right)t}$$

$$\begin{aligned} P(t) &= |\psi(t)|^2 \\ &= |\psi(0)|^2 e^{+i\left(m+i\frac{\Gamma}{2}\right)t} e^{-i\left(m-i\frac{\Gamma}{2}\right)t} \\ &= |\psi(0)|^2 e^{-\Gamma t} \end{aligned}$$

where $\Gamma = 1/\tau$.

$$\begin{aligned} g(E) &= \frac{1}{\sqrt{2\pi}} \int \psi(0) e^{-i\left(m-i\frac{\Gamma}{2}\right)t} e^{iEt}\, dt \\ &\propto \frac{1}{(E-m) - i\Gamma/2} \int e^{i\beta}\, d\beta \\ &\propto \frac{\delta(0)}{(E-m) - i\Gamma/2} \end{aligned}$$

$$\begin{aligned} \sigma &\propto |g(E)|^2 \\ &\propto \frac{1}{(E-m) + i\Gamma/2} \; \frac{1}{(E-m) - i\Gamma/2} \\ &\propto \frac{1}{(E-m)^2 + \Gamma^2/4} \end{aligned}$$