Particle Physics

SU(3) and Discrete Quantum Numbers

Problem Set 4

Due: Tuesday, March 14, 2000 at 5:00 p.m.

Problem 1

Find the SU(3) representations of a quark, anti-quark, and a gluon. That is what is the group representations of

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}?$$

How many singlet states are there that are candidates for possible resonances?

Problem 2

Prove the SU(3) version of the Wigner-Eckart Theorem. [Reference: Mesiah, *Quantum Mechanics*, Vol. II]

Problem 3

The **6** representation of SU(3) ($p = 2$, $q = 0$) can be thought of as the symmetric wave functions of quark pairs ($|uu\rangle$, $\frac{1}{\sqrt{2}}(|ud\rangle + |du\rangle)$,*etc.*). Construct the F_i and D_i matrices for this representation. Show that the F_i's obey the commutation relationships $[F_i, F_j] = if_{ijk}F_k$. Since this is a triangular representation of SU(3), $D_i \propto F_i$. What is the proportionality constant?

Problem 4

The B^0 ($\bar{b}d$ quarks) and $\overline{B}^0$ ($\bar{d}b$ quarks) mix in the same manner as the K^0 and $\overline{K}^0$. As a function of time, plot the probability of getting a positive lepton and a negative lepton from the decay of a state that started as a pure B^0 normalized to the sum. The relevant parameters are:

$$m_{B_0} = 5279.2 \text{ MeV/c}^2$$
$$\tau = 1.56 \times 10^{-12} \text{ sec}$$
$$\Delta m = 3.1 \times 10^{-10} \text{ MeV/c}^2$$

(Assume no CP violation). What is the time integrated probability of getting the positive lepton? [Reference: Perkins, Chapter 7.14]

Problem 5

CP violation can be measured in B_0 semileptonic decays. The CP violating parameter δ is defined as

$$\delta_\mu = \frac{\Gamma(B_0 \to \pi^-\mu^+\nu_\mu) - \Gamma(\bar{B}_0 \to \pi^+\mu^-\bar{\nu}_\mu)}{\Gamma(B_0 \to \pi^-\mu^+\nu_\mu) + \Gamma(\bar{B}_0 \to \pi^+\mu^-\bar{\nu}_\mu)}.$$

If you assume the $b \to u + W^-$ and assume there is no direct CP violation in the decay, find the relationship between δ and ϵ_B, the CP non-conserving parameter in the mass eigenstate wave function. If $\delta = .001$, what is $Re(\epsilon_B)$?

Problem 6

At the B-factories, the best chance to observe CP violation is in the decay $B^0 \to J/\psi + K_1$. Show that this final state is a CP=+1 eigenstate. CP violation can be observed in the comparisons of $B^0 \to J/\psi + K_{2\pi}$ and $\bar{B}^0 \to J/\psi + K_{2\pi}$ rates ($K_{2\pi}$ means the kaons which decay to 2 pions). If these B system truly conserved CP both in the propogation and the decay and the only CP violation was in the decay of the kaon system, what would the asymmetry ratio:

$$\frac{\Gamma(B^0 \to J/\psi + K_{2\pi}) - \Gamma(\bar{B}^0 \to J/\psi + K_{2\pi})}{\Gamma(B^0 \to J/\psi + K_{2\pi}) + \Gamma(\bar{B}^0 \to J/\psi + K_{2\pi})}$$

be? Now assume that the B^0's mass eigenstates are not CP eigenstates and have an $\epsilon_B = .001$ in the same manner as the kaon system. What now is the asymmetry ratio?

Problem 7

Griffiths 4.21.

Problem 8

Griffiths 4.37.