Particle Physics

Detectors, Decay Widths, and Cross Sections

Problem Set 2

Due: Tuesday, Feb. 8, 2000

The **Mathematica** notebook presented in class for Daltiz plot phase space generation can be found at

http://physuna.phs.uc.edu/~johnson/HEP/mathematica/dalitz.nb

The notebook for drawing the region of the Dalitz plot can be found in the solution to problem set 1's problem 3 at:

http://www.physics.uc.edu/~johnson/HEP/problems.sets/ps_1_p_3.nb

Please feel free to use and modify either of these notebooks as necessary for these problems.

Consider the decay

$$K^0(1270) \to K^{*+}(892)\pi^- \to K^0\pi^-\pi^+.$$

Problem 1: If the K^{*+} mass was exactly 892 MeV, show where on the Dalitz plot these events would fall. Show also what the energy distribution of the final state K^0, π^+, and π^- would be.

Instead of having a mass of exactly 892 MeV, the K^{*+} has a width of 50 MeV. This means that the probability distribution of masses for the $K^*(892)$ is given by a probability distribution known as a "Breit-Wigner" distribution:

$$\mathrm{Prob}(E) \propto \frac{\Gamma^2/4}{(E-m)^2 + \Gamma^2/4}$$

where E is the center of mass energy of the two particle system, m is the mass of the resonance, and Γ is the width (full width at half max) of the resonance.

Problem 2: Again on the Dalitz plot show where real $K^*(892)$ events would fall. (Hint: Randomly pick an E_1 and E_2 on the Dalitz plot and add a weight for that event proportional to the above probability to the tables.) Also plot the energy distributions for all three final state particles.

Problem 3: In reality, the $K^0(1270)$ decays to many things. In this decay mode, it decays via the reaction given above 28% of the time and to $K\rho$ 72% of the time. The mass and width of the ρ are 769 MeV and 151 MeV respectively. Again show a single Dalitz plot distribution of the decay that include both of these intermediate states. Show where each intermediate state appears on the plot. Again plot the energy distributions for all three final state particles. Show where the intermediate states give peaks in these plots.

Problem 4: Normally we accomodate decays in a wave function by giving the energy an imaginary part:

$$H\psi = (m - i\frac{\Gamma}{2})\psi.$$

What is the time dependent solution to this equation? Show that this leads to a probability of finding the particle after a time t of

$$P(t) = P(0)e^{-\frac{t}{\tau}}.$$

Evaluate τ. Fourier transform $\psi(t)$. This is the "propagator" for the reaction with the resonance as the intermediate state. Show that if the matrix element was proportional to this "propagator", then the cross section would have the form of the "Breit-Wigner".

Problem 5: The timing-of-flight counters in BELLE are about 60 cm from the interaction region. If the timing resolution is 25 picoseconds, up to what momentum are pions and kaons separated by more than 3 resolution times? How about kaons and protons? If the Aerogel Čerenkov counter has an index of refraction of 1.1, what is the threshold momentum for pions, kaons, and protons?

Problem 6: In PRL 74, 4376(1995), there is an illustration of an apparatus used to measure the ratio of the following decays:

$$\frac{\Gamma(K_L^0 \longrightarrow \pi^0\pi^0)}{\Gamma(K_L^0 \longrightarrow \pi^+\pi^-)}$$

a. What parts of the experiment were used to measure the K_L^0's mass and energy in the charged pion decay mode? And in the neutral mode?

b. The major backgrounds come from the following reactions:

$$\begin{aligned} K_L^0 &\longrightarrow \pi^+\pi^-\pi^0 \\ &\longrightarrow \pi^0\pi^0\pi^0 \\ &\longrightarrow \pi^+\mu^-\bar{\nu}_\mu \end{aligned}$$

How was each of these backgrounds identified and rejected in this experiment?