**Particle Physics**

**QCD and Weak Decays**

Problem Set 2

Due: May 16, 2000

**Remember – Abstracts and titles of term paper are due May 2, 2000.**

**Problem 1 – Color Counting**:

*a.* Show that a $q\bar{q}$ pair in a color singlet state cannot annihilate into a single gluon. That is, show that the color factor

$$c_{\bar{q}}^{\dagger}\lambda_i c_q$$

is zero for all $\lambda_i$ if

$$c_{\bar{q}}^{\dagger}c_q = \frac{1}{\sqrt{3}}(\bar{r}r + \bar{b}b + \bar{g}g).$$

*b.* Evaluate the color factor for the annihilation of the $\bar{q}q$ color singlet pair into two gluons. That is evalute the 8x8 matrix:

$$\sum_{i=1}^{3} c_{\bar{q}}^{\dagger}\lambda_j c_i c_i^{\dagger}\lambda_k c_q.$$

Explain the symmetry that you observe in the matrix in terms of the color state of the gluons.

*c.* Use conservation principles to show that a pseudoscalar meson can decay into two gluons (the gluons have the same discrete quantum numbers as the photon), but that a vector meson cannot. Show that vector mesons need to decay into three gluons (since one is excluded by the color argument given in part (a)). This is the origin of "Zweig's Rule"" which states that quark diagrams may not have disconnected quark lines. If those diagrams did occur, they would probablilities that were less than the connected diagrams by at least a factor of $g_s^8$.

**Problem 2 – Altarelli-Parisi Evolution Equations**

*a.* Show that

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

What assumptions do you have to make about the kinematics to get this result? (Ref: Halzen and Martin).

*b.* Given the quark structure function in a meson at a given $Q^2$ as

$$q(x) = 6x(1-x),$$

use the evolution equations to determine the structure function at $2Q^2$ and at 1/2 $Q^2$. Plot them. What assumptions did you have to make about the integrals?

**Problem 3 – Pion Decay**

In class I stated that the matrix element squared for pion decay was

$$|M|^2 = 2G_F^2 f_\pi^2 m_\mu^2 (m_\pi^2 - m_\mu^2).$$

Show this explicitly. Then show that this leads to the decay rate

$$\Gamma = \frac{G_F^2 f_\pi^2}{8\pi} m_\mu^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2.$$

**Problem 4 – Pion Decays–Continued:**

Show that the decay rate ratio

$$\frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{m_e^2}{m_\mu^2}\frac{\left(m_\pi^2 - m_e^2\right)^2}{\left(m_\pi^2 - m_\mu^2\right)^2}$$

is determined by the fact that the interaction was a vector type interaction (*i.e.*some combination of vector and axial vector) and that the V-A form is indistinguishable from a pure V or pure A in terms of this ratio. If you could measure the muon's polarization, would the number of left-handed muons differ from the number of right handed ones for V vs. V-A vs. A? If so, by how much?

**Problem 5 – Muon decays**:

The muon decay rate is given by:

$$d\Gamma = \frac{G_F^2 m_\mu^5}{3 \times 2^6 \pi^3}\left[2\epsilon^2(3-2\epsilon)\right]\left[1 + \left(\frac{1-2\epsilon}{3-2\epsilon}\right)\cos\theta\right]\left[\frac{1 - \hat{\mathbf{n}}\cdot\hat{\mathbf{s}}_{\mathbf{e}}}{2}\right] d\epsilon\frac{\sin\theta d\theta d\phi}{4\pi}$$

where $\epsilon = E_e/E_{e,max}$, $\cos\theta = \hat{\mathbf{n}}\cdot\hat{\mathbf{s}}_\mu$, $\hat{\mathbf{s}}_{\mathbf{e}}$ and $\hat{\mathbf{s}}_\mu$ are the spin directions of the electron and muon respectively, and $\hat{\mathbf{n}}$ is the direction of the outcoming electron. Compare the number of electrons going in the $+z$ hemisphere to the number going in the $-z$ hemisphere for a beam that has 100% muon polarization in $+z$. Plot result as a function of electron energy. (This is the way the muon polarization could be determined in the previous problem.)

**Problem 5 – Muon decays**:

Find the electronenergy distribution and asymmetry from polarized muon decay if the interaction is scaler-scaler, *i.e.*,

$$\mathcal{L}_{int} = G_F/\sqrt{2}\left(\bar{u}_{\nu_\mu} u_\mu\right)\left(\bar{u}_e v_{\nu_e}\right).$$