**Particle Physics**

Relativity and Accelerators

Problem Set 1

Due: Jan. 25, 2000

*Remember: Use the relativistic equations of motion for all problems.*

**Problem 1 – Two Body Decays**: A particle of mass $m_0$ decays into two particles of mass $m_1$ and $m_2$.

*a.* Find the momentum and energy of each of these particles in the rest frame of the parent particle.

*b.* If the parent particle has a momentum $p$ in the lab, find the minimum and maximum energy of particle 1 in the lab.

1. Evaluate the minimum and maximum laboratory energy for the neutrino coming from the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ for a 100 GeV/c pion beam.
2. Do the same for the neutrino from $K^+ \rightarrow \mu^+ \nu_\mu$ again with a 100 GeV/c kaon beam.

*c.* Plot the laboratory energy of the neutrino coming from the pion decay as a funtion of center of mass decay angle (the angle between the vector opposite to the laboratory velocity in the pion rest frame and and the vector of the neutrino momentum in the pion rest frame). The pion has a laboratory momentum of 100 GeV/c and decays to a muon and its neutrino.

**Problem 2 – Two Body Decays (continued)**:

*a.* In the center of mass for a spinless particle, the decays are equally probable in all directions. In terms of polar coordinates, this means that they are equally probable for equal intervals of $\phi_{cm}$ and $\cos\theta_{cm}$. Write the expression for $\sin\theta_{lab}$ for the neutrino from pion decay.

*b.* Show that, in the small angle approximation, the probability that the decay occurs at a specific laboratory angle (*i.e.*, into the laboratory phase space of $d\theta_{lab}d\phi$) is proportional to $|d\cos\theta_{cm}/d\theta_{lab}|$ (where $\theta_{lab}$ is now measured from the direction of flight of the parent particle). Plot this quantity for the neutrino from a 100 GeV pion beam and find the most probable neutrino angle.

*c.* Likewise the probability that a neutrino has a given energy in the lab is proportional to $|d\cos\theta_{cm}/dE_{lab}|$. Plot the energy distribution for the neutrinos from 100 GeV pion decay.

**Problem 3 – Three Body Decays**: A particle of mass $m_0$ decays into three particles, $m_1$, $m_2$, and $m_3$.

*a.* In the center of mass of the parent particle, find the minimum and maximum possible energy of particle 1. (Make sure you conserve both energy and momentum.)

*b.* On a graph with $E_1$ (the energy of particle 1) as the horizontal axis and $E_2$ as the vertical axis, plot the kinematically allowed region for the decay $\eta' \to \eta\pi^+\pi^-$. (The masses of the $\eta'$, $\eta$, and $\pi$ are 957.8 MeV/c$^2$, 547.3 MeV/c$^2$, and 139.6 MeV/c$^2$ respectively. In the future, masses of particles will not be given; you will need a place to look them up.) The region should fall inside the triangle bounded by $E_1 = m_\pi$, $E_2 = m_\pi$, and $E_1 + E_2 = m_{\eta'} - m_\eta$. [The triangle is purely the energy conservation constraint without any consideration given to momentum conservation.]

This plot is known as the Dalitz plot. If the decay matrix element is structureless, decays will occur with equal probability everywhere within the allowed region.

**Problem 4 – Center of Mass Energies**: Find the energy where cosmic ray protons are above threshold for the reaction $p\gamma \to pe^+e^-$ when colliding with photons from the 2.7° microwave background in the universe. Cosmic rays above this energy are expected to lose energy very quickly due to this process.

**Problem 5 – Colliders vs. Fixed Target**

Consider the reaction $pp \to npW^+$ were the mass of the $W^+$ is 80.22 GeV/c. If one of the two protons is stationary in the lab, what momentum must the other proton have to allow this reaction? If both protons had the same magnitude of momentum, but were travelling in opposite directions and collided head on, what momentum would they have to have to allow the above reaction? Calculate the $s$ variable for both of these minimum momenta.

**Problem 6 – Synchrotrons**

The Fermilab Tevatron accelerates protons to 1 TeV and bends the particles through a circular tunnel that is 1 km in diameter.

1. What magnetic field is needed in the dipole magnets if 3/4ths of the circle is filled with such magnets.
2. If the machine allows 24 bunches (24 groups of particles with the groups spaced equally around teh circumference of the ring), what is the minimum RF frequency at which the machine can run?
3. The protons are accelerated from 120 GeV/c to 1 TeV/c in 10 seconds. If ther is only one RF cavity in the machine, how much energy must the protons gain each time they transverse the cavity?
4. If, because of phase stability considerations, the bunches cross the cavity at a phase of $\pi/4$ radians, what is the peak voltage on the cavity?
5. In collider mode, the machine runs with two bunches each of protons and antiprotons travelling in opposite direction. Each bunch collides with another bunch each 1/4 circuit of the machine. For the experimentalist sitting at a fixed position around the ring, how much time is there between bunch crossings in his/her apparatus?

**Problem 7 – FDO Accelerator**: Another lattice type (other than the FODO lattice discussed in class) consists of a focusing quad placed right next to a defocusing quad and then a drift space (presumeably for dipole bending magnets). This lattice can be approximated in the thin lens approximation by a focusing lens with focal length $f$ followed by a small drift space $L_1$ then a defocusing lens with the same focal length and finally followed by a long drift space $L_2$. The structure then repeats all the way around the ring. What is the stability criterion for this lattice? For $L_1 = L_2/4$ and the shortest focal length that is stable, draw an off axis ray that transverses an orbit that repeats itself every cell.