Particle Physics

Electromagnetic Cross Sections

Problem Set 1

Due: Thursday, April 20, 2000

Problem 1 – Electrodynamics of Scalar Particles:

Make a toy model with a vector photon that interacts with a massive complex scaler field. Earlier in the year, we found that the gauge invariant Lagrangian was:

$$\mathcal{L} = (\partial_\mu \phi^\dagger - i g_e A_\mu \phi^\dagger)(\partial^\mu \phi - i g_e A^\mu \phi) - m^2 c^2 \phi^\dagger \phi + F_{\mu\nu} F^{\mu\nu}.$$

Justify the following Feymann rules for the vertices:

a. A factor $-ig_e(p + p')_\mu$ (where p and p' are the two momenta for the scalar fields) for the three point vertex (hint: what to the deriviatives do to the plane wave functions); and

b. A factor of $+2ig_e^2 g_{\mu\nu}$ for the four point vertex.

Problem 2 – Scalar - Scalar Scattering: Use the rules determined in problem 1 to calculate the electromagnetic differential cross section for a scalar particle of mass m and charge q to scatter off of another scalar particle of mass M and charge Q when the latter starts at rest in the lab. Show that if $M \to \infty$, this cross section becomes the Rutherford scattering formula. (Note: include terms only to order g_e^2 in the matrix element.)

Problem 3 – Scalar – Spinor Scattering: Show that an electron scattering off of a scalar particle ($E \gg m_e, \gg m_\phi$) has the differential cross section:

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \cos^2 \frac{\theta}{2}$$

where E is the incoming electron energy, E' is the outgoing electron energy, and

θ is the angle between the incoming and outgoing elkectron. If the scalar where a meson, how would the structure function of the meson enter? Hint: To get the standard form, express the meson current j_μ as a general function of the meson incoming and outgoing momenta. Then require current conservation at the vertex for physical meson. Reference: Bjorken and Drell, Section 10.9.

Problem 4 – Polarized Electron Positron Annihilation:

If the electron spin is included in the projection operators, they become:

$$u\bar{u} = (\gamma^\mu p_\mu + m)\frac{1 + \gamma^5\gamma^\nu s_\nu}{2}$$
$$v\bar{v} = (\gamma^\mu p_\mu - m)\frac{1 + \gamma^5\gamma^\nu s_\nu}{2}$$

Find the matrix element for an electron and positron pair with definite spins to produce a mu pair of unknown spins. Simplify this for the case when $E_e \gg m_e$. Compare the matrix elements for both spins aligned along the z axis and spins anti-aligned along the axis.

Problem 5 – Pion toy model

Let ϕ be a wave function for an SU(2) vector particle (i.e., for a pi-plus state, $|\pi^+\rangle = e^{ip^\mu x_\mu/\hbar}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, etc.). The "toy" model for this interaction with a scaler ρ is the same as that given in class except now the ϕ's are vectors of three functions rather than one function. For this model, find the ratio:

$$\frac{\Gamma(\rho \to \pi^0\pi^0)}{\Gamma(\rho \to \pi^+\pi^-)}$$

In the real world, the ρ is a vector particle (spin = 1)and cannot decay into $\pi^0\pi^0$. Why not?