

- RELABELING: Changing all occurrences of a symbol  $a$  to a new symbol not already in the presentation, interchanging all occurrences of two symbols  $a$  and  $b$ , or interchanging all occurrences of  $a$  and  $a^{-1}$  for some  $a \in S$ .
- SUBDIVIDING: Replacing every occurrence of  $a$  by  $ae$  and every occurrence of  $a^{-1}$  by  $e^{-1}a^{-1}$ , where  $e$  is a new symbol not already in the presentation.

- CONSOLIDATING: If  $a$  and  $b$  always occur adjacent to each other either as  $ab$  or  $b^{-1}a^{-1}$ , replacing every occurrence of  $ab$  by  $a$  and every occurrence of  $b^{-1}a^{-1}$  by  $a^{-1}$ , provided that the result is one or more words of length at least 3 or a single word of length 2.

- REFLECTING (Figure 6.18):

$$\langle S \mid a_1 \dots a_m, W_2, \dots, W_k \rangle \mapsto \langle S \mid a_m^{-1} \dots a_1^{-1}, W_2, \dots, W_k \rangle.$$

- ROTATING (Figure 6.19):

$$\langle S \mid a_1 a_2 \dots a_m, W_2, \dots, W_k \rangle \mapsto \langle S \mid a_2 \dots a_m a_1, W_2, \dots, W_k \rangle.$$

- CUTTING (Figure 6.20): If  $W_1$  and  $W_2$  both have length at least 2,

$$\langle S \mid W_1 W_2, W_3, \dots, W_k \rangle \mapsto \langle S, e \mid W_1 e, e^{-1} W_2, W_3, \dots, W_k \rangle.$$

- PASTING (Figure 6.20):

$$\langle S, e \mid W_1 e, e^{-1} W_2, W_3, \dots, W_k \rangle \mapsto \langle S \mid W_1 W_2, W_3, \dots, W_k \rangle.$$

- FOLDING (Figure 6.21): If  $W_1$  has length at least 3,

$$\langle S, e \mid W_1 e e^{-1}, W_2, \dots, W_k \rangle \mapsto \langle S \mid W_1, W_2, \dots, W_k \rangle.$$

We also allow  $W_1$  to have length 2, provided that the presentation has only one word.

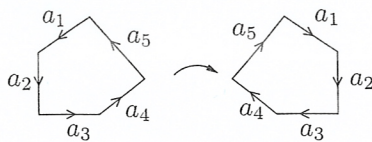


FIGURE 6.18. Reflecting.

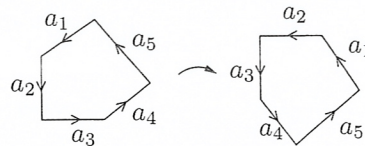


FIGURE 6.19. Rotating.

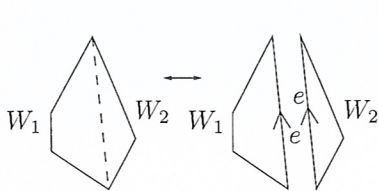


FIGURE 6.20. Cutting/pasting.

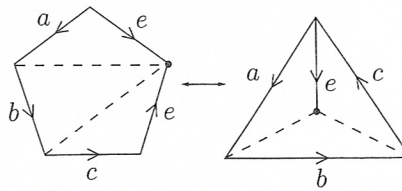


FIGURE 6.21. Folding/unfolding.