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# *LINEAR ALGEBRA REVIEW*

## I Systems of Equations

(1) General linear system of equations
- What is this?
- What constitutes a solution?
- What are the: coefficients? variables? constants?

(2) Geometric viewpoint—lines and planes

(3) Solution possibilities
- none, a unique one, infinitely many

(4) Finding solutions
- manipulation phase into upper triangular system
- back-substitution to get solutions
- row reduction via elementary row operations
- Gaussian elimination (GE) and Gauss-Jordan elimination (GJE)
- what is general solution? how to find?

(5) Row equivalent systems
- why do these have the same solutions?

(6) Homogeneous systems
- why is there always a solution?
- what is general solution? how to find?
- what is connection to non-homogeneous system?

## II Matrices

(1) Notation
- entries, columns, rows

(2) Arithmetic
- scalar multiplication and addition
- matrix multiplication
- inverses (what? how get? "big theorem")

(3) Connection with linear systems of equations
- coefficient and augmented matrices
- elementary row operations
- row equivalent matrices
- GE and row echelon form (REF)
- GJE and reduced row echelon form (RREF)

(4) Interpretation of (R)REF regarding existence & uniqueness of solutions
- the BIG theorem
- zero rows (consistency—existence of solutions)
- number of non-leading ones (arbitrary variables—uniqueness of solutions)
- number of leading ones (what?)

(5) Non-singular matrices
- definition/meaning connection with inverses
- how to calculate?
- connection with solutions to (non)-homogeneous system
- "big theorem"

(6) Transpose
- definition/meaning
- symmetric and skew-symmetric matrices
- upper and lower triangular matrices
- diagonal matrices

## III Euclidean Space

(1) $\mathbb{R}^n$
- vectors in $\mathbb{R}^n$

- arithmetic (scalar multiplication, vector addition)
- scalars vs vectors vs sets of vectors

(2) Sets of vectors
- closed wrt scalar multiplication
- closed wrt vector addition

(3) Vector subspaces of $\mathbb{R}^n$
- definition (closed wrt scalar mult and vector add)
- vector subspaces of $\mathbb{R}^2$ and $\mathbb{R}^3$
- basic example: the *null space* $\mathcal{NS}(A)$ of a matrix $A$
- what does $\mathcal{NS}(A) = \mathbb{R}^n$ mean?
- what does $\mathcal{NS}(A) = \{0\}$ mean?

(4) Linear combinations
- definition/meaning
- how to tell if a vector is a LC of other vectors?
- reinterpretation of vector subspace definition
  ($\mathcal{V}$ is a vector subspace if it is closed wrt LC)
- interpretation of matrix multiplication
  ($AX$ is a LC of the columns of $A$)

(5) The span of a set of vectors
- definition/meaning
- how to tell if a vector is in the span of other vectors?
- basic example: the *column space* $\mathcal{CS}(A)$ of a matrix $A$
- other interpretation of $\mathcal{CS}(A)$ (all $B$ st $AX = B$ consistent)
- what does $\mathcal{CS}(A) = \mathbb{R}^m$ mean?
- what does $\mathcal{CS}(A) = \{0\}$ mean?

(6) Linear independence vs linear dependence
- definition/meaning (especially for 2 or 3 vectors)
- how to tell when vectors LI?
- how to find LI subset of set of vectors?
- connection with solutions to homogeneous system of equations
- "big theorem"

(7) Basis
- definition/meaning
- minimal spanning set vs maximal linearly independent set
- how to find (especially for $\mathcal{CS}(A)$ and $\mathcal{NS}(A)$)
- special case when (?) get spanning set iff linearly independent

(8) Rank and nullity
- definition
- connection with existence and uniqueness of solutions
  what does it mean if
  the rank of $A$ is: $0$ ? $m$? $< m$?
  the nullity of $A$ is: $0$ ? $n$? $> 0$?
- rank-nullity theorem
- what does this say about existence and uniqueness of solutions especially when $m > n$ or $m < n$ or $m = n$?
- why is the rank of $A^T$ the same as the rank of $A$?

(9) Coordinates
- definition/geometric meaning
- coordinate vector
- how to find coordinates?
- change of basis coordinate change matrix
  what is it? how to find it? properties
  how to use to find coordinates?

(10) Scalar or dot product
- definition/properties – algebraic and geometric
- connection with – angles, length

(11) Orthogonality
- $X \perp Y$ if ..., $X \perp \mathcal{V}$ if ..., $\mathcal{W} \perp \mathcal{V}$ if ...

- orthogonal *set* of vectors (LI or LD?)
- orthonormal *set* of vectors (LI or LD?)

(12) Vector projection
- definition
- properties

(13) Gram-Schmidt algorithm
- what it does, how to do it, what it tells you

(14) Orthogonal complement
- definition and properties of $\mathcal{V}^\perp$
- how to determine $\mathcal{V}^\perp$
- $\dim \mathcal{V}^\perp = ?$
- Why is $\mathbb{R}^n = \mathcal{V} + \mathcal{V}^\perp$? Why is $\mathbb{R}^n = \mathcal{V} \oplus \mathcal{V}^\perp$?

(15) Orthogonal projection
- definition
- how to calculate
- geometric interpretation
- closest pt idea