Orthogonal Projection

Linear Algebra
MATH 2076

UNIVERSITY OF Cincinnati

Orthogonal Complement

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Definition (Orthogonal Complement of a Set)

The *orthogonal complement* of a non-empty set W of vectors in $\mathbb{R}^n$ is

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Here $\vec{w}$ is the 'part' of $\vec{x}$ that is parallel to $\mathbb{W}$ and $\vec{z}$ is the 'part' of $\vec{x}$ that is orthogonal to $\mathbb{W}$. How do we find $\vec{w}$ and $\vec{z}$?

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Let $\vec{u}$ be a fixed vector, and $\vec{x}$ a variable vector.

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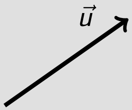

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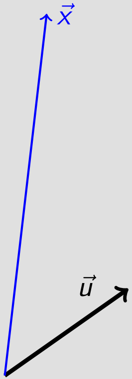

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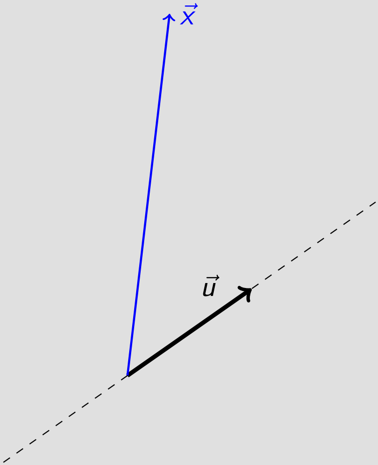

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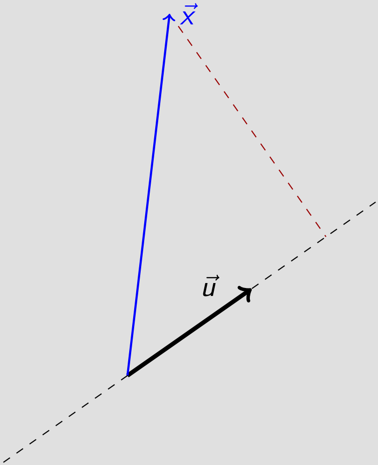

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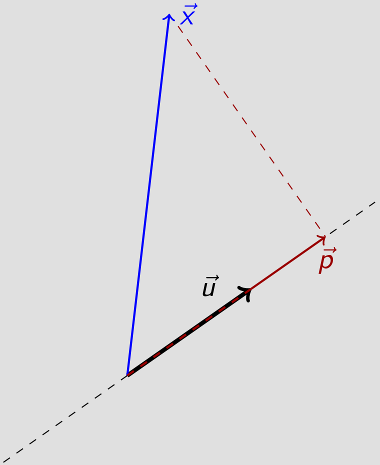

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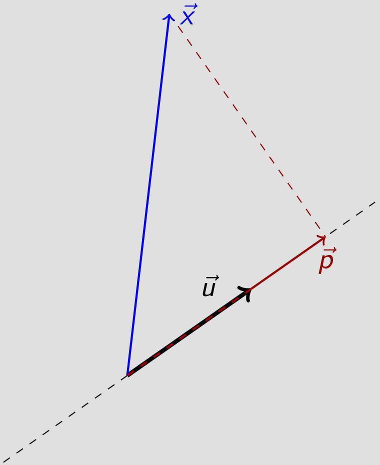

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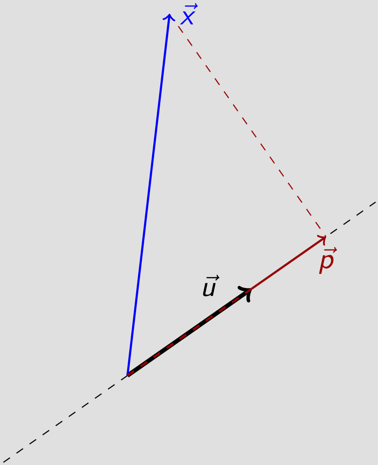

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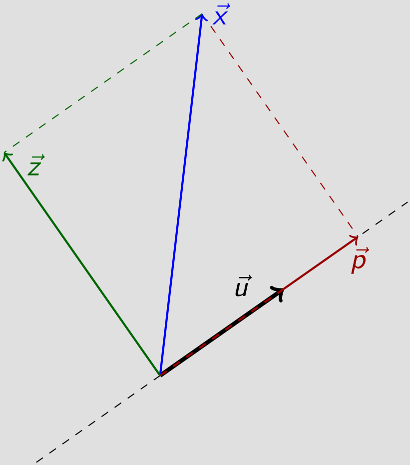

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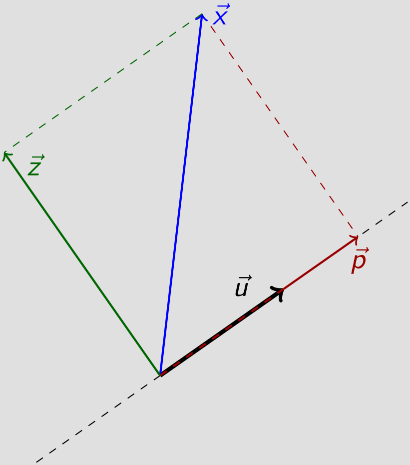

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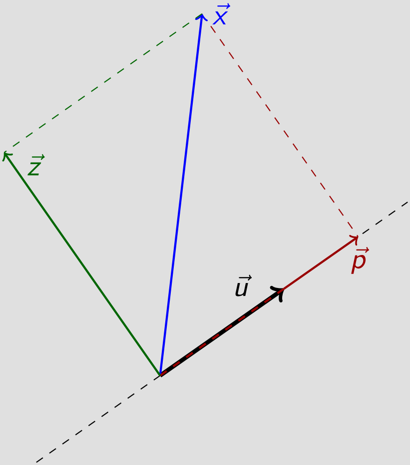

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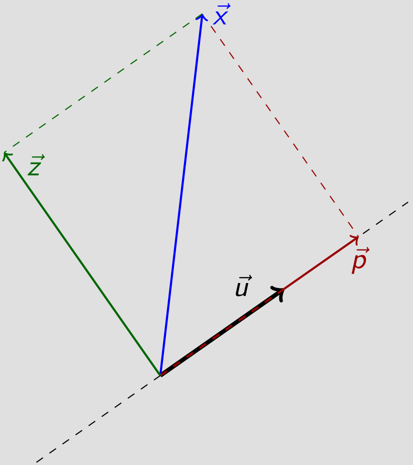

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$$s = \frac{\vec{x} \cdot \vec{u}}{\vec{u} \cdot \vec{u}}$$

and thus

$$\vec{p} = \text{Proj}_{\vec{u}}(\vec{x}) = \frac{\vec{x} \cdot \vec{u}}{\vec{u} \cdot \vec{u}}\,\vec{u}.$$

Example of Orthogonal Projection Onto a Vector

$\vec{0}$

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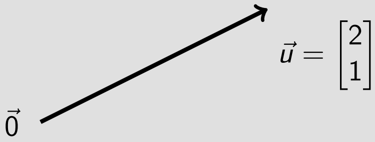

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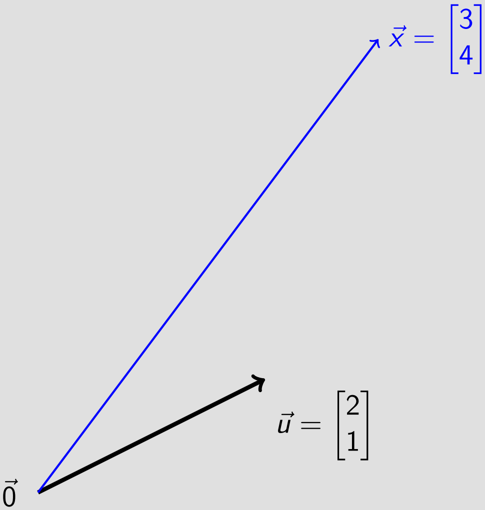

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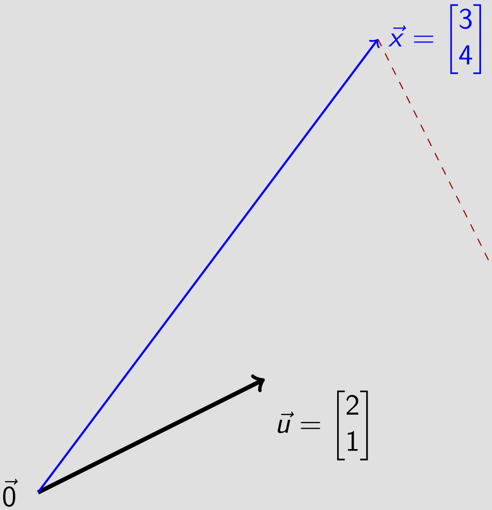

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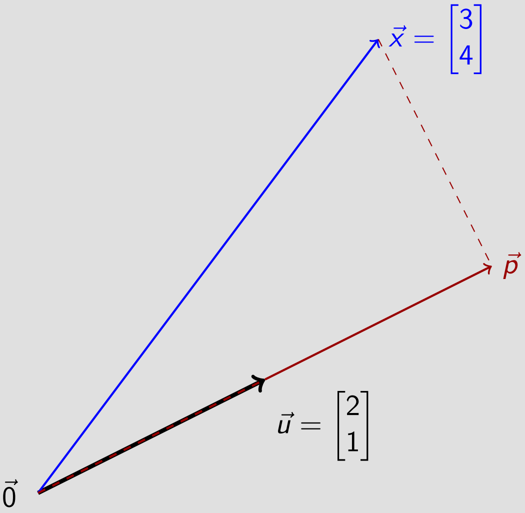

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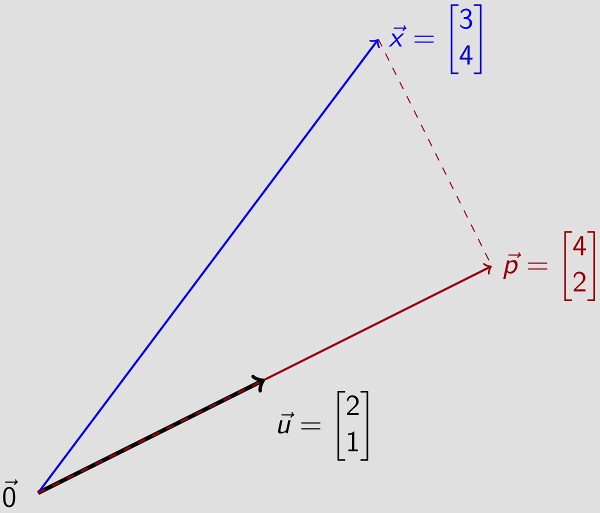

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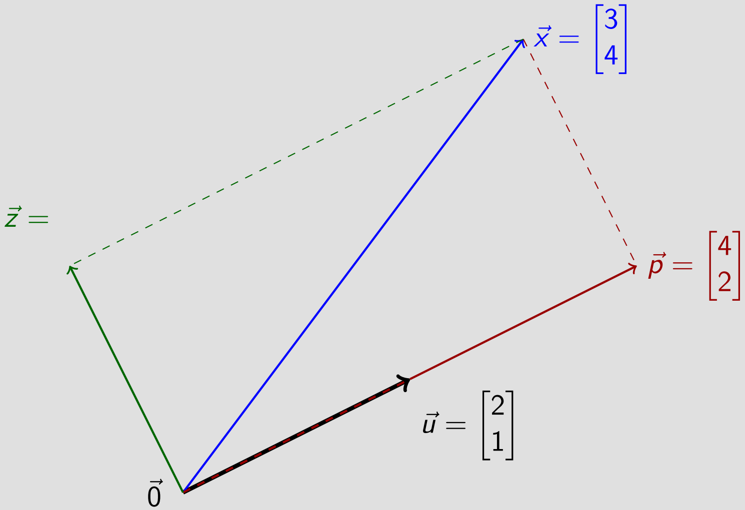

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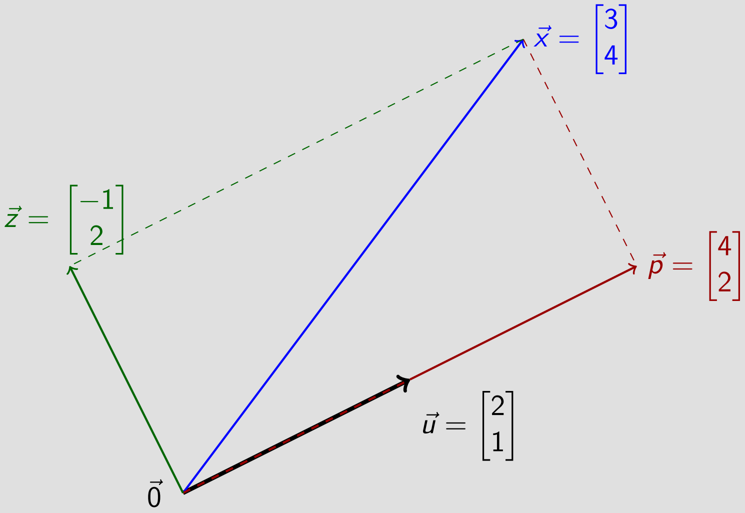

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To find c_j, look at

$$\vec{v} \cdot \vec{b}_j = \sum_{i=1}^{k} c_i \vec{b}_i \cdot \vec{b}_j = c_j \, \vec{b}_j \cdot \vec{b}_j$$

so $c_j = \dfrac{\vec{v} \cdot \vec{b}_j}{\vec{b}_j \cdot \vec{b}_j}$.

Coordinates Relative to an Orthogonal Basis

Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_k\}$ be an ortho basis for a vector subspace $\mathbb{W}$ of $\mathbb{R}^n$, so $\vec{b}_i \cdot \vec{b}_j = 0$ when $i \neq j$.

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Definition

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Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbb{W} = \mathcal{S}pan\{\vec{w}_1, \vec{w}_2\}$ where $\vec{w}_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \vec{w}_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.

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Write $\vec{x} = \vec{p} + \vec{z}$ with $\vec{p}$ in $\mathbb{W}$ and $\vec{z} \perp \mathbb{W}$, and find $\text{dist}(\vec{x}, \mathbb{W})$.

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Thus

$$\vec{p} = \vec{p}_1 + \vec{p}_2 = \frac{3}{10}\vec{w}_1 + \frac{1}{2}\vec{w}_2 =$$

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Write $\vec{x} = \vec{p} + \vec{z}$ with $\vec{p}$ in $\mathbb{W}$ and $\vec{z} \perp \mathbb{W}$, and find $\text{dist}(\vec{x}, \mathbb{W})$.

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$$\vec{p}_1 = \text{Proj}_{\vec{w}_1}(\vec{x}) = \frac{\vec{x} \cdot \vec{w}_1}{\vec{w}_1 \cdot \vec{w}_1} \vec{w}_1 = \frac{9}{30} \vec{w}_1 = \frac{3}{10} \vec{w}_1$$

and

$$\vec{p}_2 = \text{Proj}_{\vec{w}_2}(\vec{x}) = \frac{\vec{x} \cdot \vec{w}_2}{\vec{w}_2 \cdot \vec{w}_2} \vec{w}_1 = \frac{3}{6} \vec{w}_2 = \frac{1}{2} \vec{w}_2.$$

Thus

$$\vec{p} = \vec{p}_1 + \vec{p}_2 = \frac{3}{10} \vec{w}_1 + \frac{1}{2} \vec{w}_2 = \frac{1}{5} \begin{bmatrix} -2 \\ 10 \\ 1 \end{bmatrix}$$

Let $\vec{x} = \begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\mathbb{W} = \mathcal{S}pan\{\vec{w}_1, \vec{w}_2\}$ where $\vec{w}_1 = \begin{bmatrix}2\\5\\-1\end{bmatrix}, \vec{w}_2 = \begin{bmatrix}-2\\1\\1\end{bmatrix}$.
Write $\vec{x} = \vec{p} + \vec{z}$ with $\vec{p}$ in $\mathbb{W}$ and $\vec{z} \perp \mathbb{W}$, and find $\operatorname{dist}(\vec{x}, \mathbb{W})$.

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Thus

$$\vec{p} = \vec{p}_1 + \vec{p}_2 = \frac{3}{10}\vec{w}_1 + \frac{1}{2}\vec{w}_2 = \frac{1}{5}\begin{bmatrix}-2\\10\\1\end{bmatrix} \quad \text{and} \quad \vec{z} = \vec{x} - \vec{p} = \frac{7}{5}\begin{bmatrix}1\\0\\2\end{bmatrix}.$$

Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbb{W} = \mathcal{S}pan\{\vec{w}_1, \vec{w}_2\}$ where $\vec{w}_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \vec{w}_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.
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Finally $\text{dist}(\vec{x}, \mathbb{W}) = \|\vec{x} - \vec{p}\| = \|\vec{z}\| = \frac{7}{5}\sqrt{5} = \frac{7}{\sqrt{5}}$.

Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \vec{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \vec{w}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \vec{w}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

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Let $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \vec{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \vec{w}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \vec{w}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. Find $\text{Proj}_{\mathbb{W}}(\vec{x})$ where $\mathbb{W} = \mathcal{S}pan\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$. Since $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is an orthog basis for $\mathbb{W}$, $\vec{p} = \text{Proj}_{\mathbb{W}}(\vec{x}) = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$ where $\vec{p}_i = \text{Proj}_{\vec{w}_i}(\vec{x})$ for $i = 1, 2, 3$. We compute

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The *reflection of $\vec{x}$ across $\mathbb{W}$* is given by

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