Similar Matrices and Diagonalization

Linear Algebra
MATH 2076

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$$A = PDP^{-1} \quad \text{where} \quad P = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \ldots \vec{v}_n \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \vdots & 0 \\ 0 & 0 & \ldots & \lambda_n \end{bmatrix}.$$

A 2×2 Example

The matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ has *simple* eigenvalues $\lambda_1 = 5$ and $\lambda_2 = -1$ with assoc'd eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

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There is **no** eigenbasis assoc'd with A_3, so A_3 is **not** diagonalizable.

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$$A\vec{x} = A\big(c_1\vec{v}_1 + c_2\vec{v}_2\big) = c_1 A\vec{v}_1 + c_2 A\vec{v}_2 = 5c_1\vec{v}_1 - c_2\vec{v}_2$$

which says that

$$\left[A\vec{x}\right]_{\mathcal{B}} = \begin{bmatrix} 5c_1 \\ -c_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} \left[\vec{x}\right]_{\mathcal{B}}.$$

Thus using $\mathcal{B}$-coordinates, the action of A is just multiplication by the diagonal matrix

$$D = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

Recall that $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ has *simple* eigenvalues $\lambda_1 = 5$ and $\lambda_2 = -1$ with assoc'd eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Since A's two eigenvectors are LI, they form an *eigenbasis* $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$.

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But, how do we get $A\vec{x}$?

Let's draw a picture for the coordinate mapping $\vec{x} \mapsto \left[\vec{x}\right]_{\mathcal{B}}$, and also for the inverse map too.

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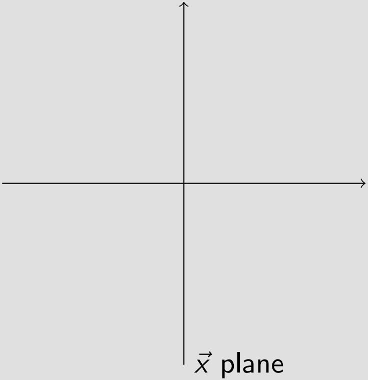

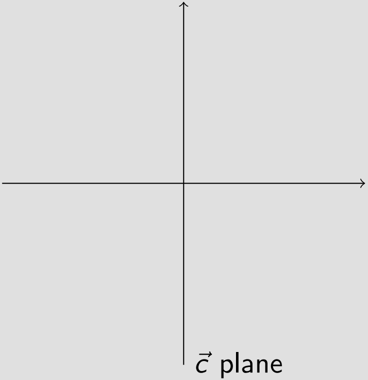

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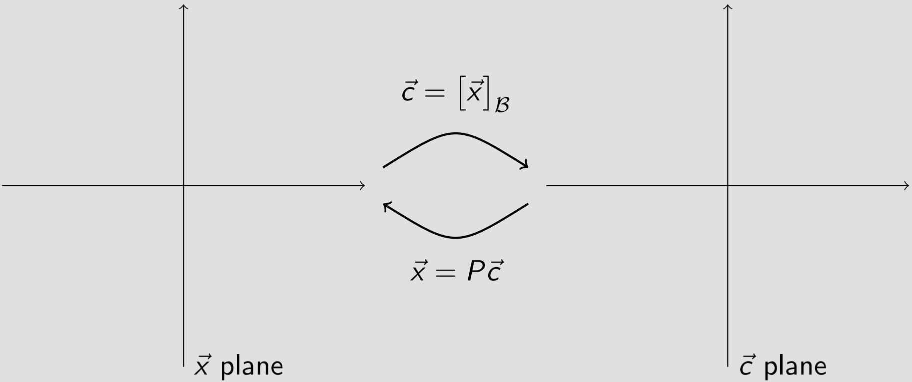

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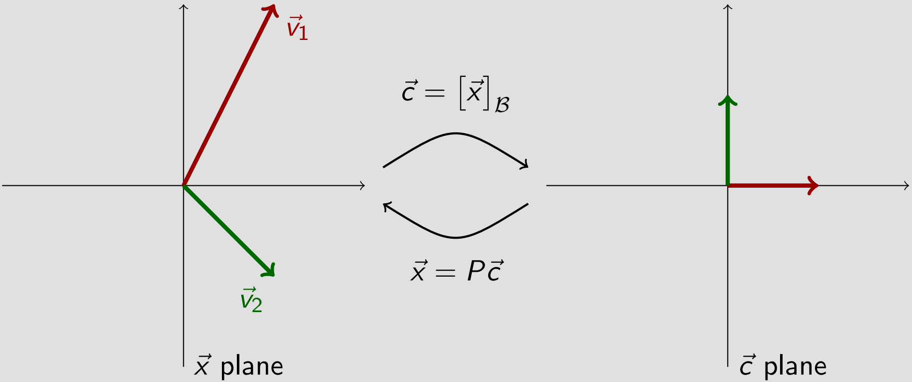

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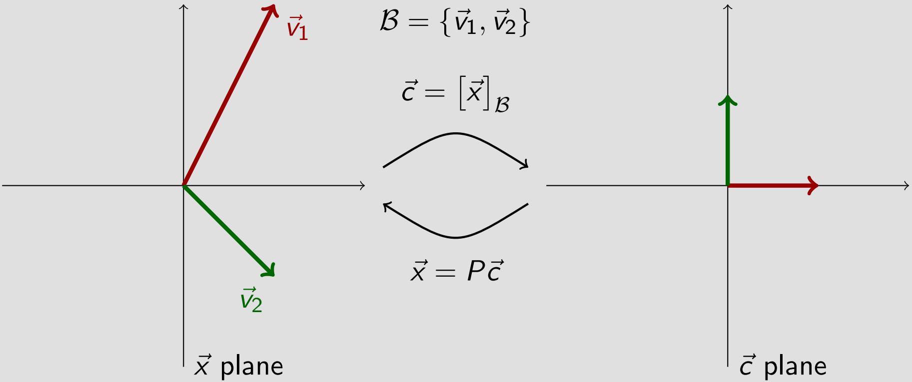

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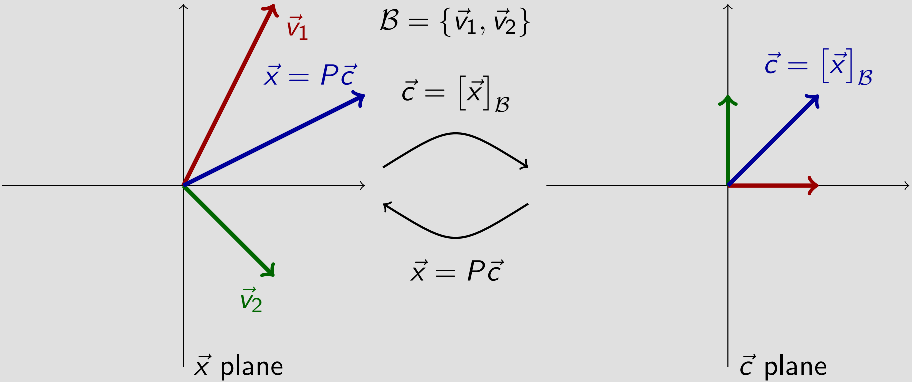

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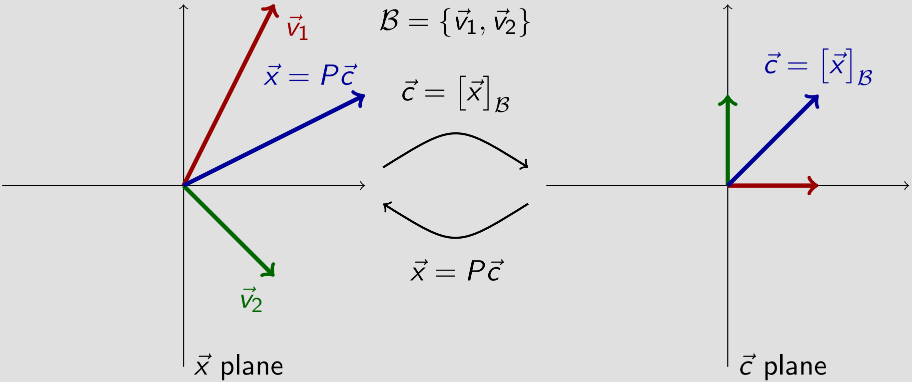

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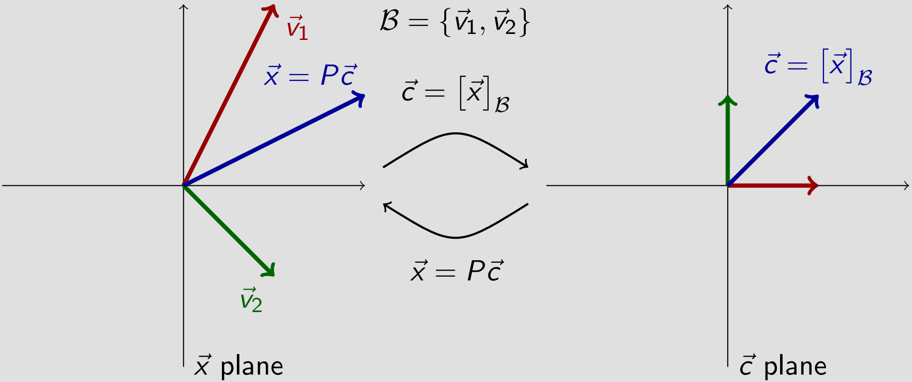

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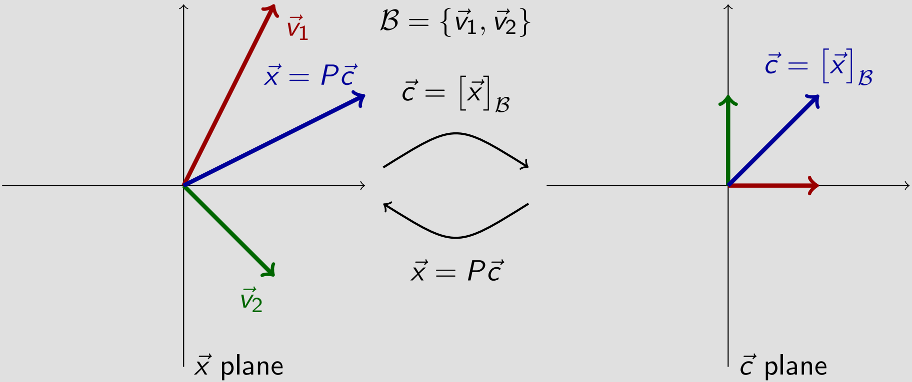

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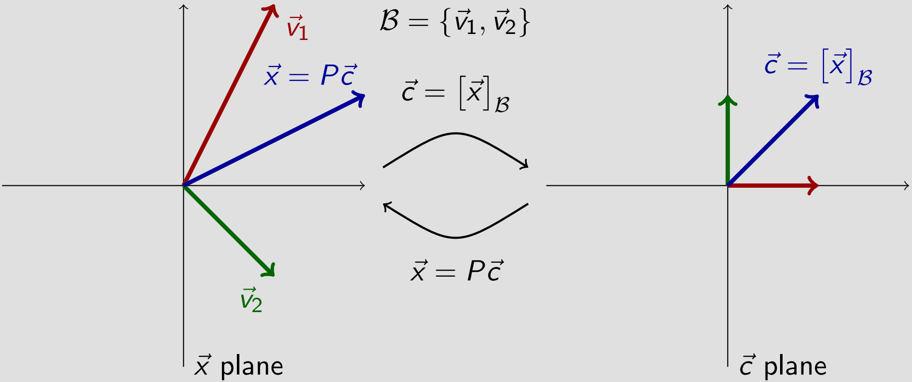

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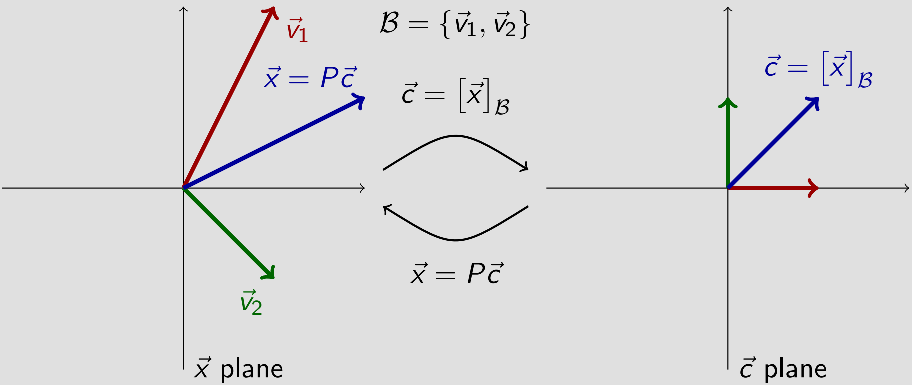

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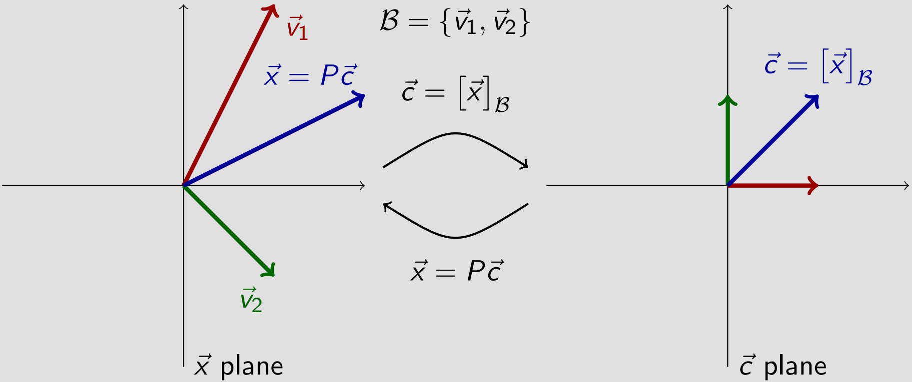

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Recall that for any vector $\vec{w}$ in $\mathbb{R}^2$ we have

$$\vec{w} = P\left[\vec{w}\right]_{\mathcal{B}} \quad \text{and} \quad \left[\vec{w}\right]_{\mathcal{B}} = P^{-1}\vec{w}$$

where the $\mathcal{B}$ to $\mathcal{S}$ change of coordinates matrix P is given by

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Thus

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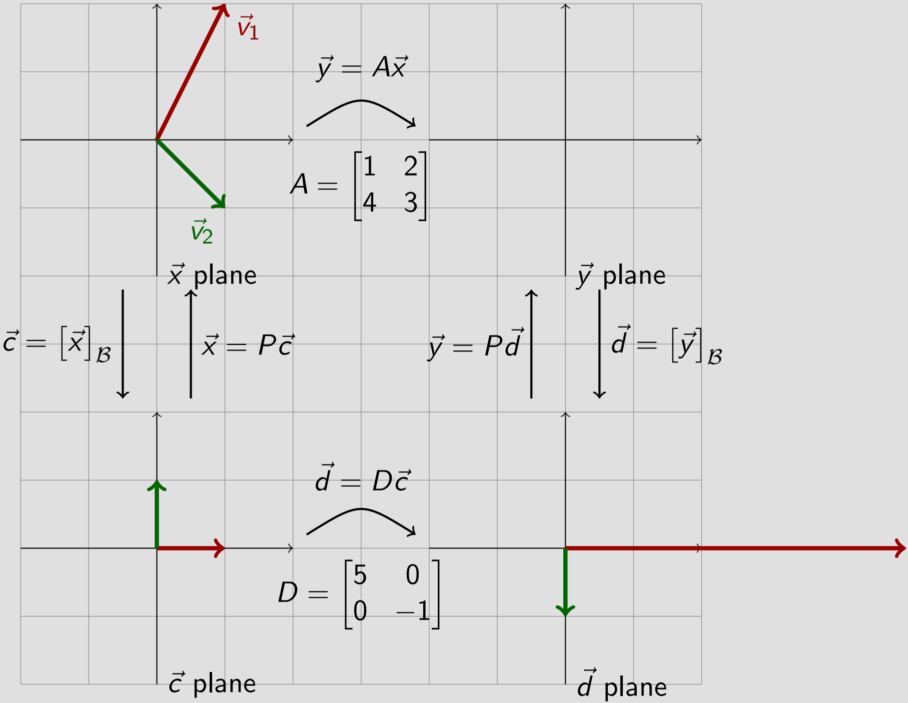
Thus A and D are *similar* matrices. How to "see" the LT $\vec{x} \mapsto A\vec{x}$?



$\vec{v}_1$
$\vec{y} = A\vec{x}$
$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$
$\vec{v}_2$
$\vec{x}$ plane
$\vec{y}$ plane
$\vec{c} = [\vec{x}]_{\mathcal{B}}$
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