Characteristic Equation and Similar Matrices

Linear Algebra
MATH 2076

UNIVERSITY OF Cincinnati

EigenVectors, EigenValues, EigenSpaces

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For item (3), just note that on $\mathbb{E}(\lambda)$, A acts like the dilation $A\vec{x} = \lambda\vec{x}$ (since each *non-zero* vector in $\mathbb{E}(\lambda)$ is an eigenvector for A).

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Notice that A's two eigenvectors $\vec{v}_1, \vec{v}_2$ are LI, so form an *eigenbasis*.

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Characteristic Polynomials and Equations

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A 3×3 Matrix with Three *Simple* Eigenvalues

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There is **NO** eigenbasis assoc'd with A_3.

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When we have an *eigenbasis* assoc'd with A, it is especially simple to understand the action of the matrix transformation $\vec{x} \mapsto A\vec{x}$.

This is because A is *similar* to a diagonal matrix!

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Recall that $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ has *simple* eigenvalues $\lambda_1 = 5$ and $\lambda_2 = -1$ with assoc'd eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Since A's two eigenvectors are LI, they form an *eigenbasis* $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$.

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Look at

$$A\vec{x} = A\big(c_1\vec{v}_1 + c_2\vec{v}_2\big) = c_1A\vec{v}_1 + c_2A\vec{v}_2 = 5c_1\vec{v}_1 - c_2\vec{v}_2$$

which says that

$$\big[A\vec{x}\big]_{\mathcal{B}} = \begin{bmatrix} 5c_1 \\ -c_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} \big[\vec{x}\big]_{\mathcal{B}}.$$

Thus using $\mathcal{B}$-coordinates, the action of A is just multiplication by the diagonal matrix

$$D = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

But, how do we get $A\vec{x}$?

From the previous slide: WTF $A\vec{x}$ and we know

$$\left[A\vec{x}\right]_{\mathcal{B}} = D\left[\vec{x}\right]_{\mathcal{B}} \quad \text{where} \quad D = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

and we have an eigenbasis assoc'd with A given by

$$\mathcal{B} = \{\vec{v}_1, \vec{v}_2\} \quad \text{where} \quad \vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

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Recall that for any vector $\vec{w}$ in $\mathbb{R}^2$ we have

$$\vec{w} = P\left[\vec{w}\right]_{\mathcal{B}} \quad \text{and}$$

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Thus

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So,

$$A = PDP^{-1} \text{ where } D = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \; P = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}.$$

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We say that A and D are *similar* matrices.