EigenVectors, EigenValues, EigenSpaces

Linear Algebra
MATH 2076

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For $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$, $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

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See the geogebra file Eigen2x2Ex1gbg.

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What do items (3) and (4) above tell us?

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So, $\mathbb{E}(3) = \mathcal{S}pan\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ and $\mathbb{E}(2) = \mathcal{S}pan\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$.

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Note that on $\mathbb{E}(2)$, A acts like the dilation $\vec{x} \mapsto A\vec{x} = 2\vec{x}$.

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The matrix $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$ has an eigenvalue $\lambda = 2$ and $\mathbb{E}(2)$ is the plane in $\mathbb{R}^3$ given by $\mathbb{E}(2) = \mathcal{S}pan\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$.

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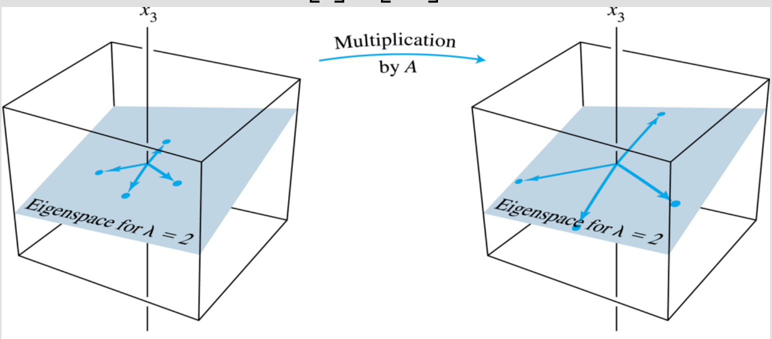

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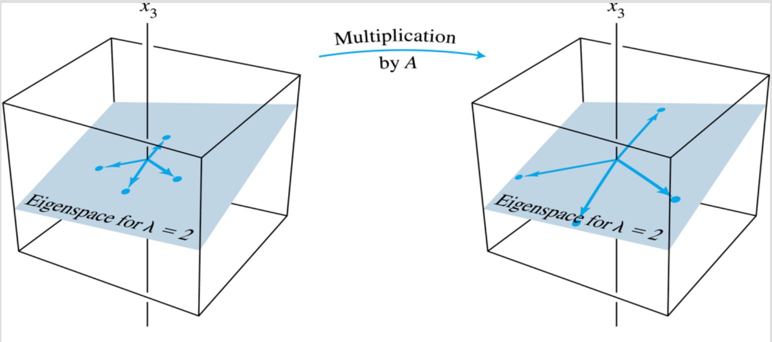

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See Eigen3x3Ex1.ggb

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Notice that the two eigenvectors for A are LI, so form a basis.