The Matrix of a Linear Transformation

Linear Algebra
MATH 2076

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i.e., $\big[T(\vec{v})\big]_\mathcal{A} = M\big[\vec{v}\big]_\mathcal{B}$. We call M the *matrix for T relative to $\mathcal{B}$ and $\mathcal{A}$*

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Picture for the matrix for T relative to $\mathcal{B}$ and $\mathcal{A}$

We have a linear transformation $\mathbb{V} \xrightarrow{T} \mathbb{W}$ and bases $\mathcal{B}, \mathcal{A}$ for $\mathbb{V}, \mathbb{W}$ resp.

$$\mathbb{V} \xrightarrow{\quad T \quad} \mathbb{W}$$

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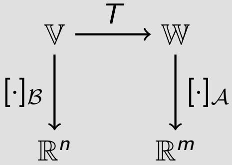
Consider the $\mathcal{B}$ and $\mathcal{A}$ coord maps $\mathbb{V} \to \mathbb{R}^n$ and $\mathbb{W} \to \mathbb{R}^m$.



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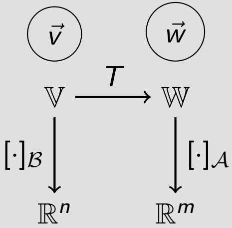

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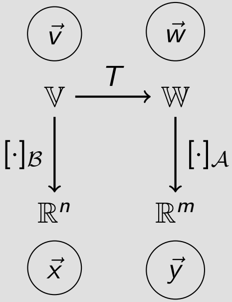
$$\vec{x} = \left[\vec{v}\right]_{\mathcal{B}} \text{ and } \vec{y} = \left[\vec{w}\right]_{\mathcal{A}}.$$



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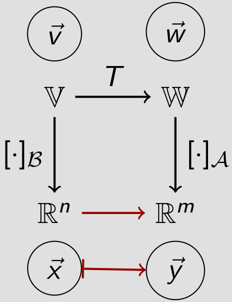
$$\vec{x} = \left[\vec{v}\right]_{\mathcal{B}} \text{ and } \vec{y} = \left[\vec{w}\right]_{\mathcal{A}}.$$

Consider the LT $\mathbb{R}^n \to \mathbb{R}^m$ given by $\vec{x} \mapsto \vec{y}$.



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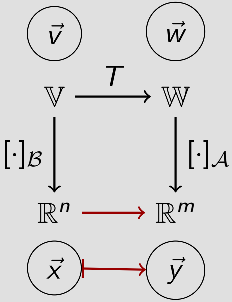
Consider the LT $\mathbb{R}^n \to \mathbb{R}^m$ given by $\vec{x} \mapsto \vec{y}$. This is a matrix transformation, and



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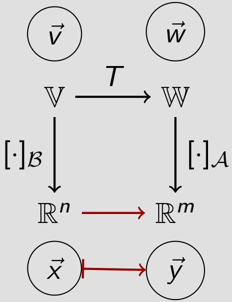
Consider the LT $\mathbb{R}^n \to \mathbb{R}^m$ given by $\vec{x} \mapsto \vec{y}$. This is a matrix transformation, and

$$\vec{y} = M\vec{x}$$



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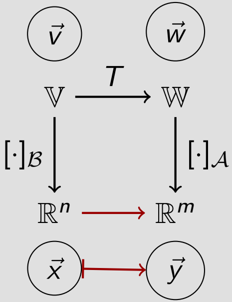

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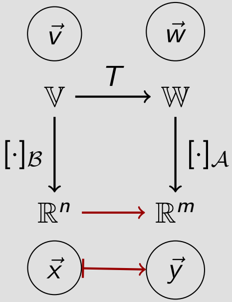
Consider the LT $\mathbb{R}^n \to \mathbb{R}^m$ given by $\vec{x} \mapsto \vec{y}$. This is a matrix transformation, and $\big[T(\vec{v})\big]_{\mathcal{A}} = \big[\vec{w}\big]_{\mathcal{A}} = \vec{y} = M\vec{x}$



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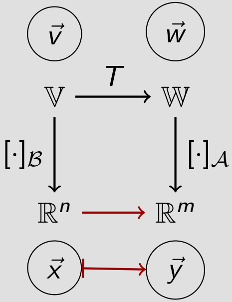

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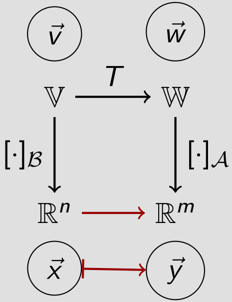
Consider the LT $\mathbb{R}^n \to \mathbb{R}^m$ given by $\vec{x} \mapsto \vec{y}$. This is a matrix transformation, and $\big[T(\vec{v})\big]_{\mathcal{A}} = \big[\vec{w}\big]_{\mathcal{A}} = \vec{y} = M\vec{x} = A\big[\vec{v}\big]_{\mathcal{B}}$ where $M = \big[T\big]_{\mathcal{A}\mathcal{B}}$.



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You can remember this as $\left[T\right]_{\mathcal{AB}} = \left[T(\mathcal{B})\right]_{\mathcal{A}}$, but this abuses notation!

The $\mathcal{B}$-Matrix for a Linear Transformation

Suppose $\mathbb{V} = \mathbb{R}^n = \mathbb{W}$ and $\mathcal{B} = \mathcal{A}$, so $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^n$ is a linear transformation and $\mathcal{B}$ is some basis for $\mathbb{R}^n$.

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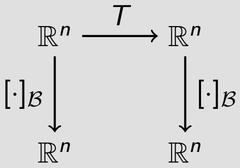

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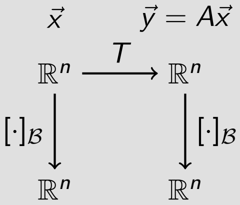

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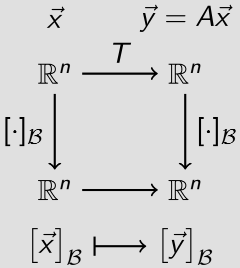

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$$\begin{array}{ccc} \vec{x} & & \vec{y} = A\vec{x} \\ \mathbb{R}^n & \xrightarrow{T} & \mathbb{R}^n \\ {\scriptstyle [\cdot]_{\mathcal{B}}}\downarrow & & \downarrow{\scriptstyle [\cdot]_{\mathcal{B}}} \\ \mathbb{R}^n & \longrightarrow & \mathbb{R}^n \\ \big[\vec{x}\big]_{\mathcal{B}} & \longmapsto & \big[\vec{y}\big]_{\mathcal{B}} \end{array}$$

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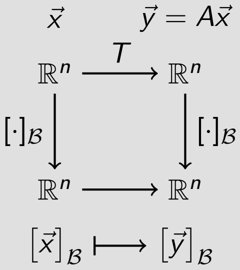

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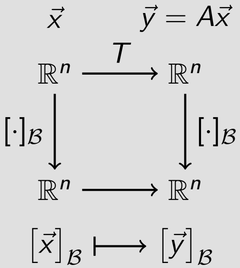

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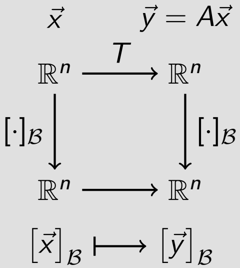

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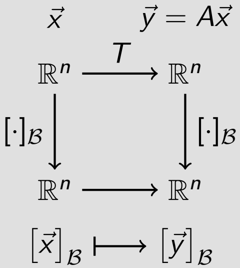

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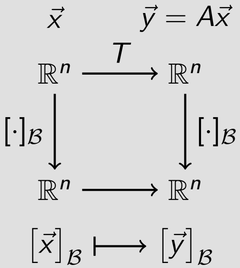
$$=$$



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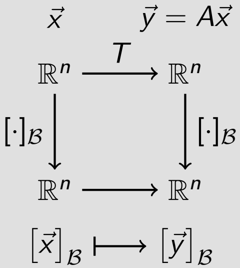

The $\mathcal{B}$-Matrix for a Linear Transformation

Suppose $\mathbb{V} = \mathbb{R}^n = \mathbb{W}$ and $\mathcal{B} = \mathcal{A}$, so $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^n$ is a linear transformation and $\mathcal{B}$ is some basis for $\mathbb{R}^n$. Here we write $\big[T\big]_{\mathcal{B}} = \big[T\big]_{\mathcal{B}\mathcal{B}}$ for the matrix for T relative to $\mathcal{B}$ and $\mathcal{B}$, and call this the *the $\mathcal{B}$-matrix for T*.

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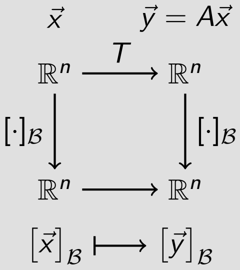

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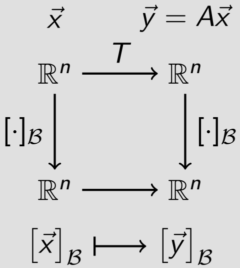

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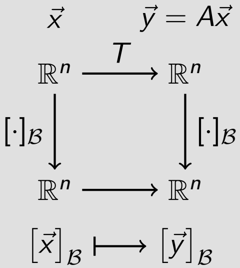

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$$\begin{array}{ccc} \vec{x} & & \vec{y} = A\vec{x} \\ \mathbb{R}^n & \xrightarrow{T} & \mathbb{R}^n \\ {\scriptstyle [\cdot]_{\mathcal{B}}}\downarrow & & \downarrow{\scriptstyle [\cdot]_{\mathcal{B}}} \\ \mathbb{R}^n & \longrightarrow & \mathbb{R}^n \\ \left[\vec{x}\right]_{\mathcal{B}} & \longmapsto & \left[\vec{y}\right]_{\mathcal{B}} \end{array}$$

It follows that the $\mathcal{B}$-matrix for T is given by $\left[T\right]_{\mathcal{B}} = P^{-1}AP$, so $A = P\left[T\right]_{\mathcal{B}}P^{-1}$. Thus $A = \left[T\right]_{\mathcal{E}}$ and $\left[T\right]_{\mathcal{B}}$ are *similar* matrices!

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This says that $D = \left[T\right]_{\mathcal{B}} = \left[T\right]_{\mathcal{B}\mathcal{B}}$.

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