Coordinate Vectors and the Coordinate Mapping

Linear Algebra
MATH 2076

$\mathcal{B}$-Coordinate Vectors and the $\mathcal{B}$-Coordinate Mapping

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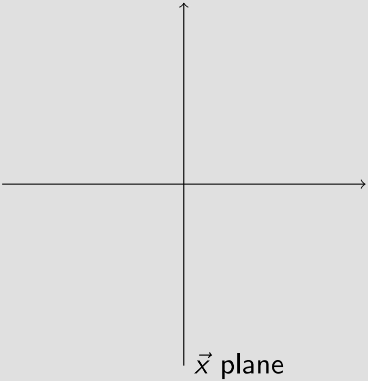

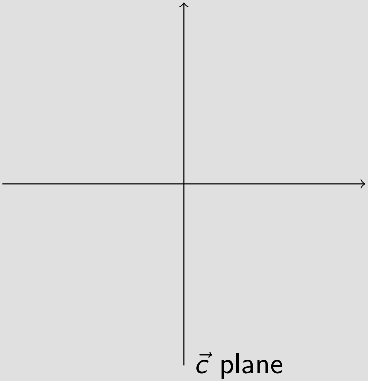

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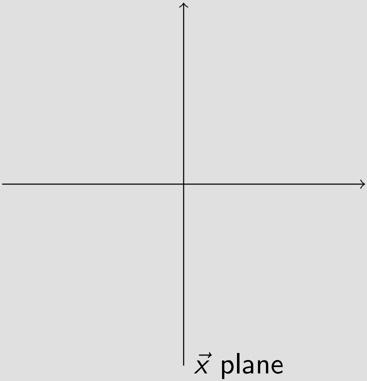

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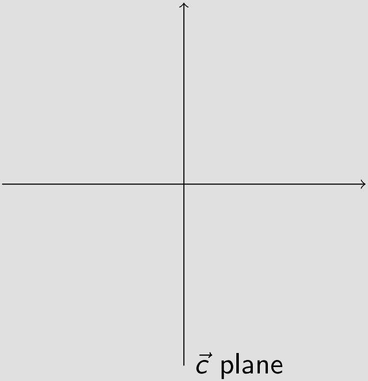

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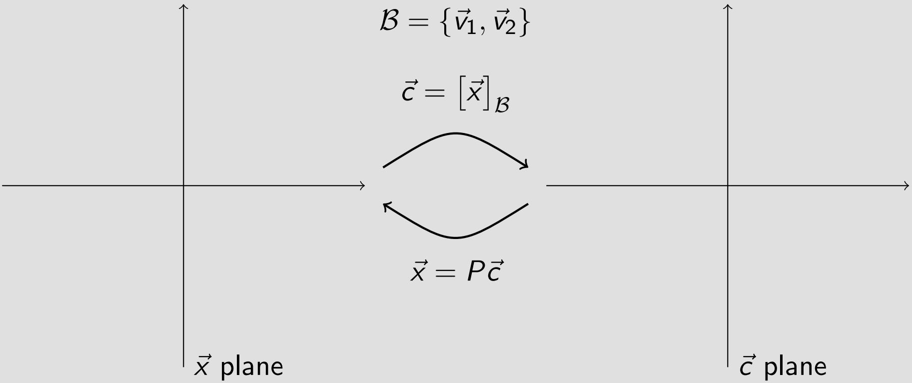

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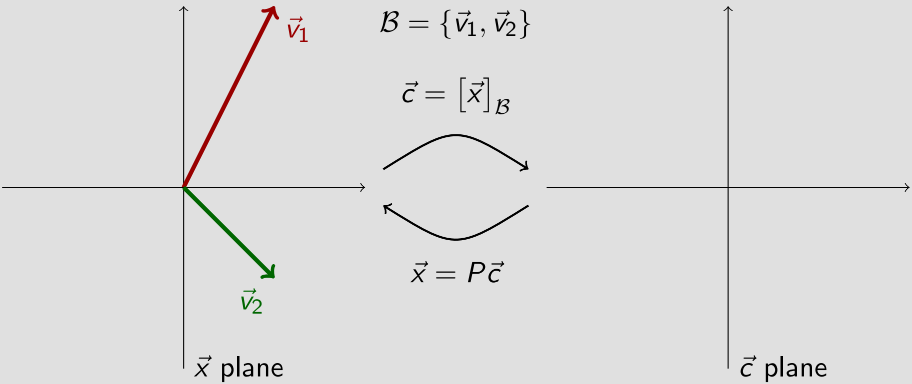

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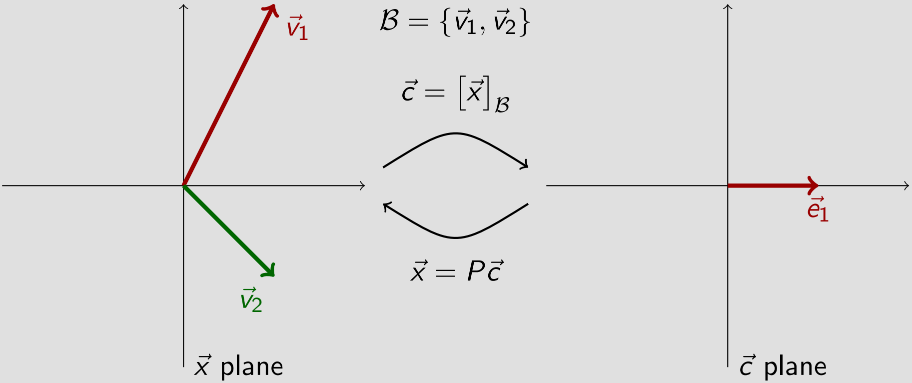

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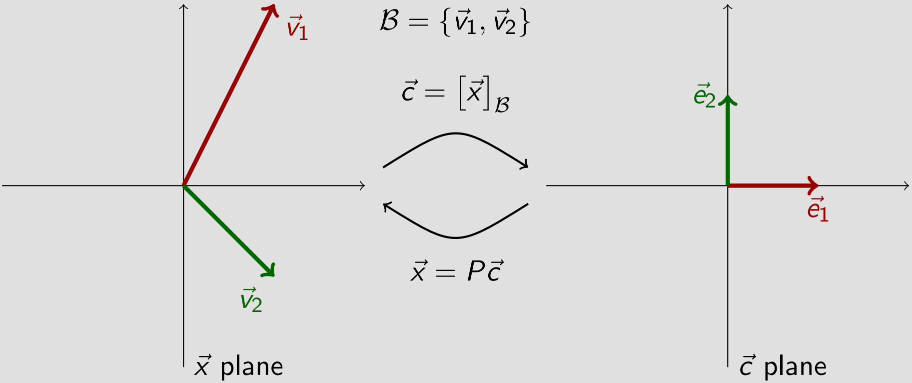

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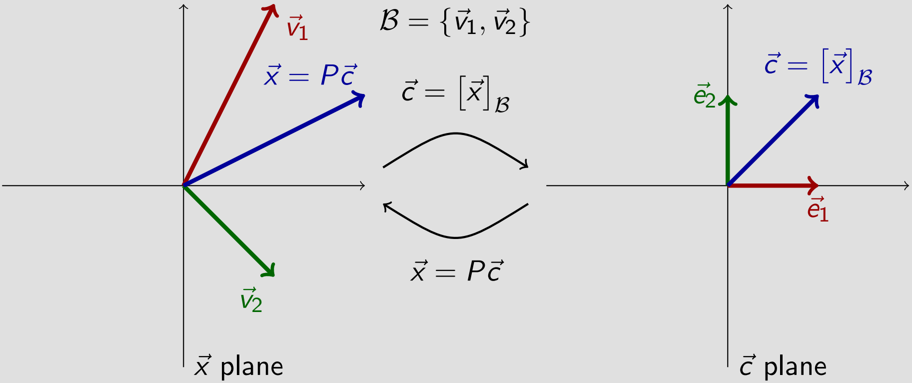

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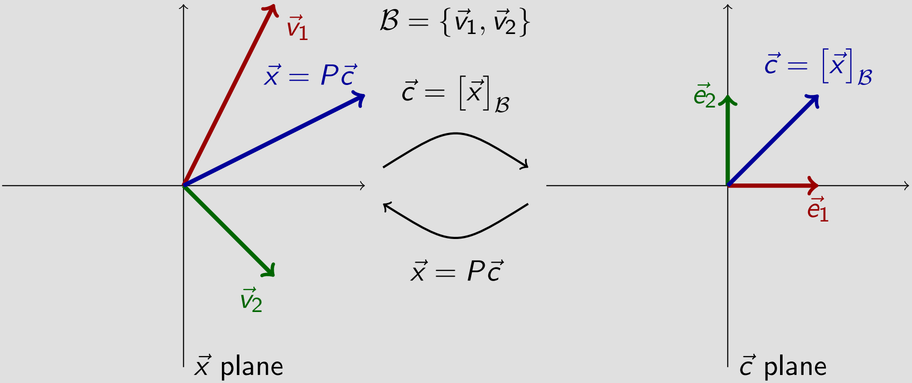

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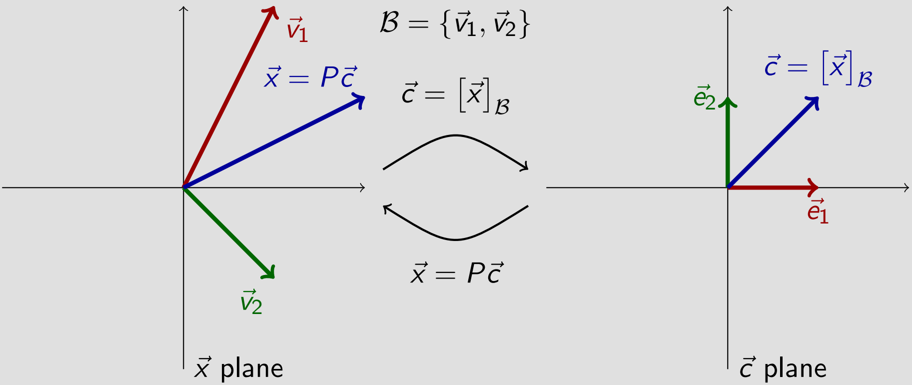

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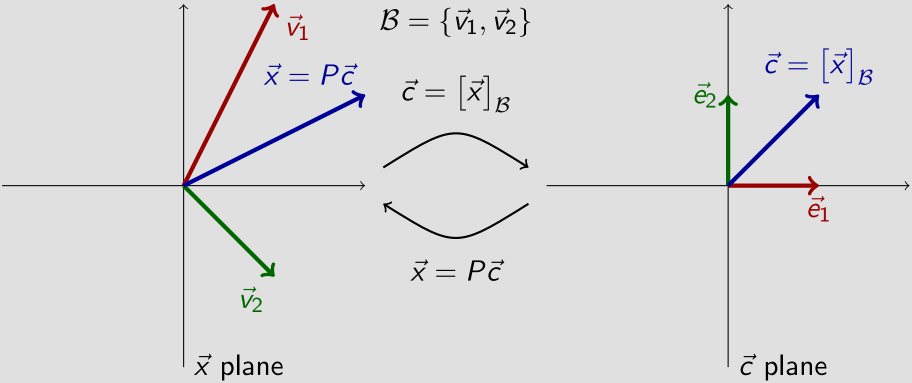

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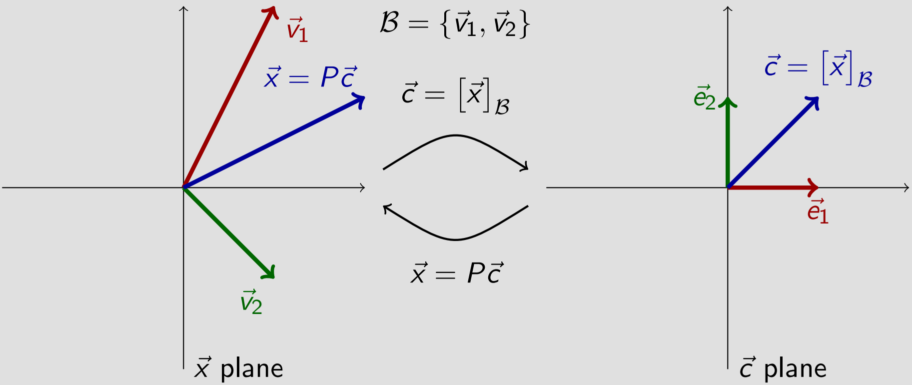

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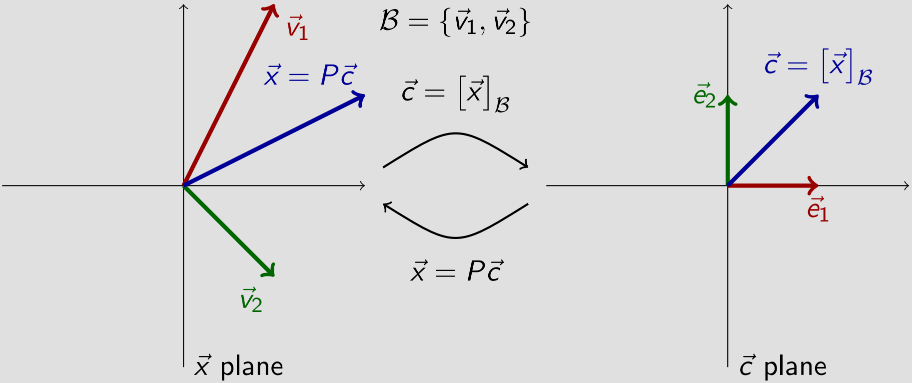

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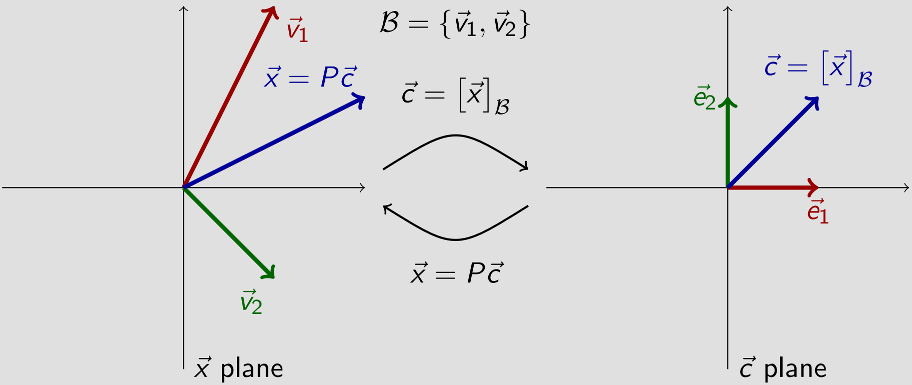
$$P = P_{\mathcal{E}\mathcal{B}} = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix} =$$

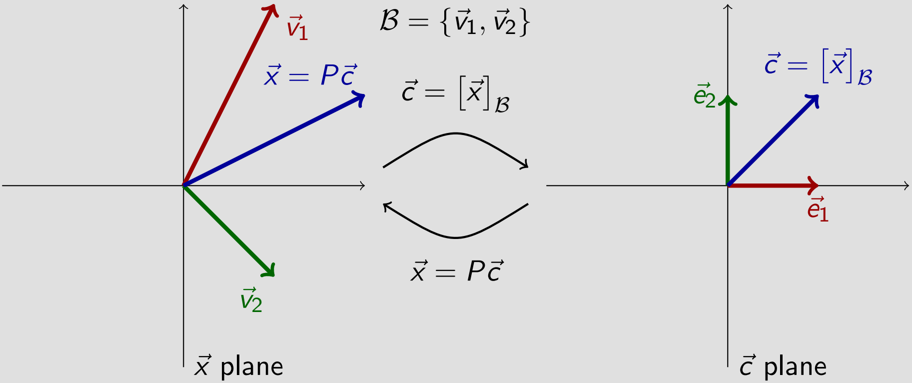
Let's draw a picture for the coordinate mapping $\vec{x} \mapsto \vec{c} = [\vec{x}]_{\mathcal{B}}$, and also for the inverse map too.



Here $\vec{c} = [\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ means $\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2 = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix} \vec{c} = P[\vec{x}]_{\mathcal{B}}$; so the $\mathcal{B}$ to $\mathcal{E}$ change of coordinates matrix P is given by

$$P = P_{\mathcal{E}\mathcal{B}} = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

Let's draw a picture for the coordinate mapping $\vec{x} \mapsto \vec{c} = [\vec{x}]_{\mathcal{B}}$, and also for the inverse map too.



Here $\vec{c} = [\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ means $\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2 = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix} \vec{c} = P[\vec{x}]_{\mathcal{B}}$; so the $\mathcal{B}$ to $\mathcal{E}$ change of coordinates matrix P is given by

$$P = P_{\mathcal{E}\mathcal{B}} = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \quad \text{and then} \quad [\vec{x}]_{\mathcal{B}} = P^{-1}\vec{x}.$$