Bases and Coordinates

Linear Algebra
MATH 2076

Coordinates and Coordinate Vectors

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Note that $\left[\vec{v}\right]_{\mathcal{B}}$ is a vector in $\mathbb{R}^p$.

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that is, $\vec{v} = 3\vec{a}_1 + 2\vec{a}_2$. Note that $\vec{v}$ is in $\mathbb{R}^3$ whereas $\left[\vec{v}\right]_{\mathcal{B}}$ is in $\mathbb{R}^2$.

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Again, while $\vec{v}$ is a vector in $\mathbb{R}^n$, $\left[\vec{v}\right]_{\mathcal{B}}$ is a vector in $\mathbb{R}^p$.

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This is also what tells us that the $\mathcal{B}$-coord mapping $\mathbb{V} \xrightarrow{[\cdot]_{\mathcal{B}}} \mathbb{R}^p$ is a linear transformation.