Linear Transformations Between Vector Spaces

Linear Algebra MATH 2076



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An important question is to know which vectors \vec{b} are in the range of T. That is: Given \vec{b} in \mathbb{W} , when can we find an \vec{x} in \mathbb{V} with $T(\vec{x}) = \vec{b}$?

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for all \vec{u} , \vec{v} in \mathbb{R}^n and all scalars s.

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$$T(s_1\vec{v}_1 + s_2\vec{v}_2 + \cdots + s_p\vec{v}_p) = s_1T(\vec{v}_1) + s_2T(\vec{v}_2) + \cdots + s_pT(\vec{v}_p)$$

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or more simply—using "summation" notation—

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This is called the *linearity principle* .



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$$\mathcal{K}er(T) = \mathcal{NS}(A)$$
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Examples of Linear Transformations

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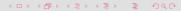
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Next, define $\mathbb{R}^{n\times n} \xrightarrow{S} \mathbb{R}^{n\times n}$ by $T(X) = X - X^T$; you should check that S is an LT. What are $\mathcal{K}er(S)$ and $\mathcal{R}ng(S)$?

Let A be a fixed $m \times n$ matrix. Define $\mathbb{R}^{n \times p} \xrightarrow{T} \mathbb{R}^{m \times p}$ by T(X) = AX; T is a linear transformation. Without knowing more about A, we cannot say much about $\mathcal{K}er(T)$ or $\mathcal{R}ng(T)$.

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Section 4.2.B

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If S(X) = 0, then $X = X^T$; so Ker(S) is the subspace of all *symmetric* $n \times n$ matrices.

HW: Show $\mathcal{R}ng(S)$ is the subspace of all *skew-symmetric* $n \times n$ matrices.

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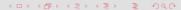
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Can we find some other poly p so that p' = D(p) = q? Sure! We just anti-differentiate, right? The poly

$$\mathbf{p}(t) = c_0 t + \frac{c_1}{2} t^2 + \frac{c_2}{3} t^3 + \dots + \frac{c_n}{n+1} t^{n+1}$$

has the property that p' = D(p) = q. This means that $\mathcal{R}ng(D) = \mathbb{P}$.

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HW: Show that $\mathcal{R}ng(D) = \mathbb{P}_{n-1}$.

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HW: Show that DS - SD = I where D is "differentiation" (as described 2 slides above) and I is the identity transformation $L(\mathbf{p}) = \mathbf{p}$.