Linear Transformations Between Vector Spaces

Linear Algebra
MATH 2076

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An important question is to know which vectors $\vec{b}$ are in the range of T. That is: Given $\vec{b}$ in $\mathbb{W}$, when can we find an $\vec{x}$ in $\mathbb{V}$ with $T(\vec{x}) = \vec{b}$?

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$$\mathcal{K}er(T) = \mathcal{NS}(A) \quad \text{and} \quad \mathcal{R}ng(T) = \mathcal{CS}(A).$$

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Now define $\mathbb{R}^{m\times n} \xrightarrow{T} \mathbb{R}^{n\times m}$ by $T(X) = X^T$; recall that X^T is the *transpose* of the matrix X given by $\left[X^T\right]_{ij} = \left[X\right]_{ji}$. Then T is an LT. What are $\mathcal{K}er(T)$ and $\mathcal{R}ng(T)$? This is easy, right?

Next, define $\mathbb{R}^{n\times n} \xrightarrow{S} \mathbb{R}^{n\times n}$ by $T(X) = X - X^T$; you should check that S is an LT.

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HW: Show $\mathcal{R}ng(S)$ is the subspace of all *skew-symmetric* $n \times n$ matrices.

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has the property that $\boldsymbol{p}' = D(\boldsymbol{p}) = \boldsymbol{q}$. This means that $\mathcal{R}ng(D) = \mathbb{P}$.

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HW: Show that $\mathcal{R}ng(D) = \mathbb{P}_{n-1}$.

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One More Example of a Linear Transformation

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HW: Show that $DS - SD = I$ where D is "differentiation" (as described 2 slides above) and I is the identity transformation $I(\boldsymbol{p}) = \boldsymbol{p}$.