A Hyperplane in $\mathbb{P}_3$

Linear Algebra
MATH 2076

The Problem and Our Strategy

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- coordinate maps "preserve" all linear combinations.

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To check for LI, we could look at $s_1\boldsymbol{q}_1 + s_2\boldsymbol{q}_2 + s_3\boldsymbol{q}_3 = \boldsymbol{0}$ and explain why this means that $s_1 = s_2 = s_3 = 0$.

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We look at the matrix $\begin{bmatrix} [\boldsymbol{q}_1]_{\mathcal{P}} & [\boldsymbol{q}_2]_{\mathcal{P}} & [\boldsymbol{q}_3]_{\mathcal{P}} \end{bmatrix}$.

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The polynomials $\boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{q}_3$ are LI iff $\left[\boldsymbol{q}_1\right]_{\mathcal{P}}, \left[\boldsymbol{q}_2\right]_{\mathcal{P}}, \left[\boldsymbol{q}_3\right]_{\mathcal{P}}$ are LI. To see if these $\mathcal{P}$-coordinate vectors are LI, we look at the matrix

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which has the indicated REF. Thus $\boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{q}_3$ <u>are</u> LI, so $\mathcal{B} = \{\boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{q}_3\}$ is a basis for $\mathbb{W}$.

Therefore, dim $\mathbb{W} = 3$; $\mathbb{W}$ is a 3-plane (aka, a hyperplane) in $\mathbb{P}_3$.

$\mathbb{W}$ is all polynomials $\boldsymbol{p}$ in $\mathbb{P}_3$ that satisfy $\boldsymbol{p}(2) = 0$

Finding the $\mathcal{B}$-coordinate Vector for $\boldsymbol{p}$

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Now we're looking at vectors in $\mathbb{R}^4$, and we know how to solve this vector equation.

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Remembering what $[\boldsymbol{q}_1]_{\mathcal{P}}, [\boldsymbol{q}_2]_{\mathcal{P}}, [\boldsymbol{q}_3]_{\mathcal{P}}$ are, we get the augmented matrix

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and then some elem row ops produce the indicated REF.

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$c_1 = 3, c_2 = -4, c_3 = 1$ and therefore $\left[\boldsymbol{p}\right]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$.

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