Properties of Determinants Using Elementary Row Operations

Linear Algebra
MATH 2076

Determinants and Elementary Row operations

The following are allowable elementary row operations.

1. Add a multiple of one row to another.
2. Multiply one row by a *non-zero* constant k.
3. Interchange two rows.

How do these elem row ops change the determinant?

Let A be a square matrix; so $\det(A) = \sum_{j=1}^{n}(-1)^{i+j} a_{ij} \det(M_{ij})$.
Suppose we perform an elem row op on A to get B. Then:

- $\det(B) = \det(A)$ for a type (1) elem row op ($\ddot\smile$)
- $\det(B) = k \det(A)$ for a type (2) elem row op
- $\det(B) = -\det(A)$ for a type (3) elem row op

Example

Find the determinant of

$$B = \begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 3 & 4 \\ 3 & 0 & 0 & 1 & 2 \\ 0 & 2 & 1 & 1 & 1 \end{bmatrix}.$$

Not a lotta zeroes anywhere :-(. Let's do some row reducing!

$$\det(B) = \det\begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 3 & 4 \\ 3 & 0 & 0 & 1 & 2 \\ 0 & 2 & 1 & 1 & 1 \end{bmatrix} = -\det\begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -6 & -8 & 2 \\ 0 & 0 & 0 & -7 & -10 \end{bmatrix}$$

$$= -\det\begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 & 20 \\ 0 & 0 & 0 & -7 & -10 \end{bmatrix}$$

$$= -\big(-40 + 140\big) = -100\,.$$