Determinants—an Introduction

Linear Algebra
MATH 2076

Example

Find the determinant of

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 & 3 \\ 3 & 4 & 1 & 0 & -1 \\ 6 & 4 & 2 & 1 & -2 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\det(A) = \det \begin{bmatrix} 1 & 2 & -1 & 0 & 3 \\ 3 & 4 & 1 & 0 & -1 \\ 6 & 4 & 2 & 1 & -2 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} = -\det \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= -\left(-\det \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix} + \det \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 1 \\ 0 & 1 & 1 \end{bmatrix} \right)$$

$$= -(-12 - 6) = +18$$