Subspaces of Euclidean Space $\mathbb{R}^n$

Linear Algebra
MATH 2076

UNIVERSITY OF Cincinnati

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- $\mathbb{V} = \Big\{$all $\vec{v}$ in $\mathbb{R}^4$ with third coordinate -7$\Big\}$ is not a subspace

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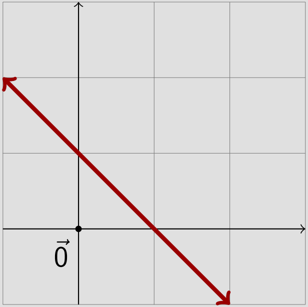

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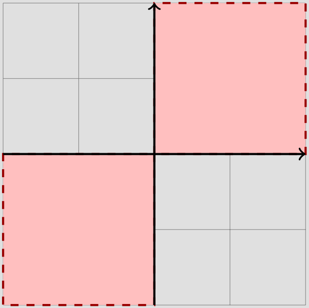

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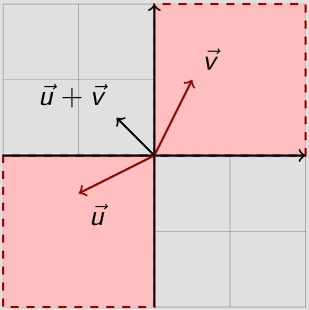

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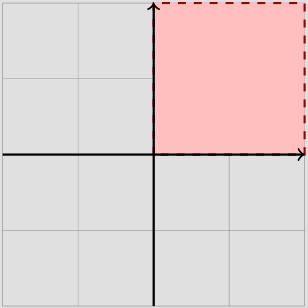

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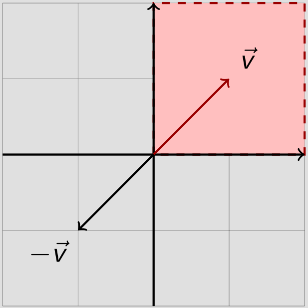

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$$\vec{u} + \vec{v} = (s_1 + t_1)\vec{v}_1 + (s_2 + t_2)\vec{v}_2 + \cdots + (s_p + t_p)\vec{v}_p$$

which is a vector in $\mathbb{V}$.

Vector Subspaces—Basic Example

Recall that a collection $\mathbb{V}$ of vectors (in $\mathbb{R}^n$) is a *vector subspace* (of $\mathbb{R}^n$) if and only if

- $\vec{0}$ is in $\mathbb{V}$,
- $\mathbb{V}$ *closed with respect to vector addition* ($\vec{u}, \vec{v}$ in $\mathbb{V} \implies \vec{u} + \vec{v}$ in $\mathbb{V}$)
- $\mathbb{V}$ *closed with respect to scalar mult* (s scalar, $\vec{v}$ in $\mathbb{V} \implies s\vec{v}$ in $\mathbb{V}$)

Let $\mathbb{V} = \mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$. Let's show that $\mathbb{V}$ is closed wrt vector addition. Let $\vec{u}, \vec{v}$ be vectors in $\mathbb{V}$. This means there are scalars $s_1, s_2, \ldots, s_p$ and $t_1, t_2, \ldots, t_p$ with

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Homework: Show that $\mathbb{V}$ is closed wrt scalar multiplication.

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Just saw that any $\mathbb{V} = \mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$ is both closed wrt vector addition and closed wrt scalar multiplication.

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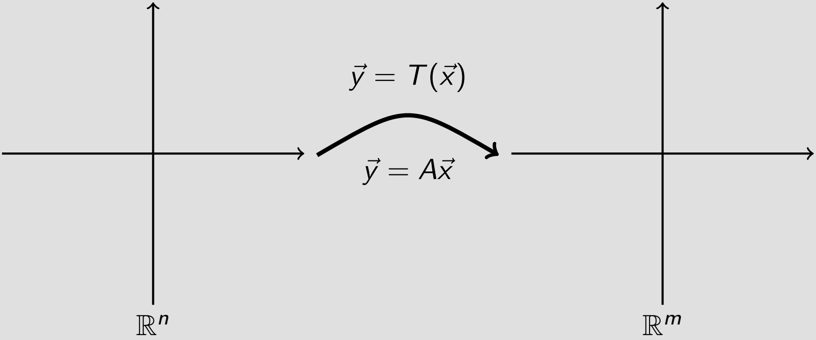

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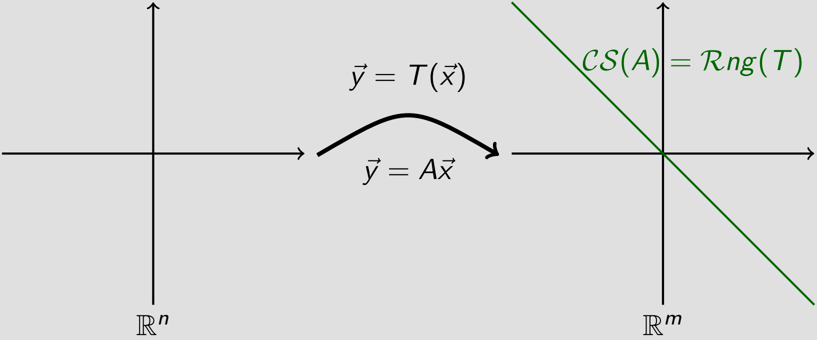

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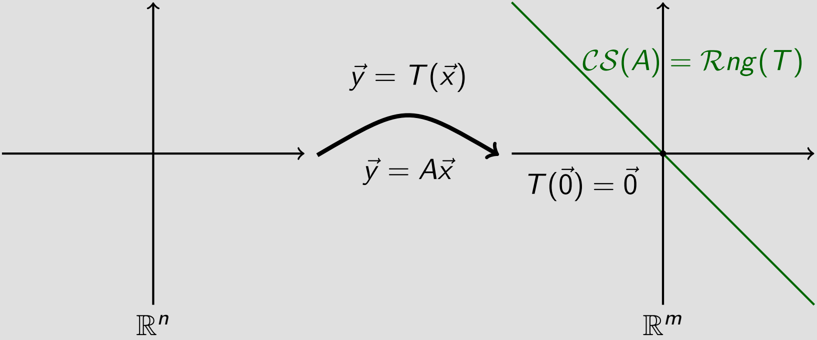

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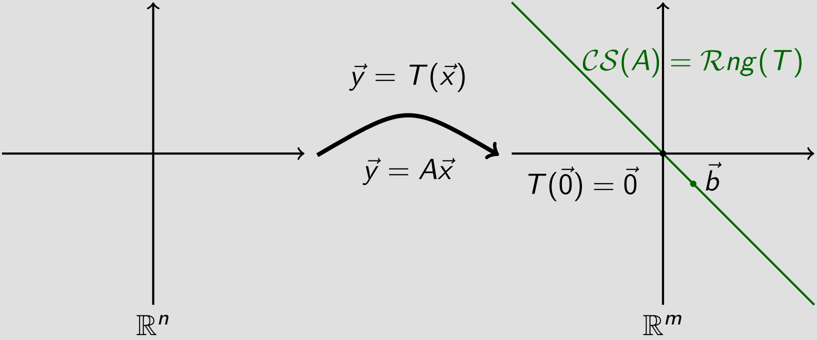

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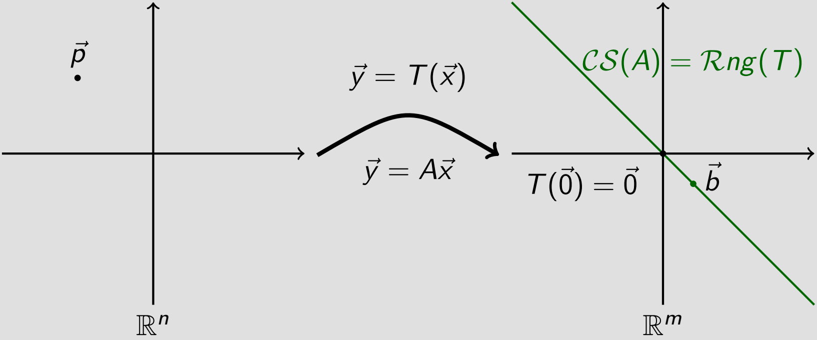

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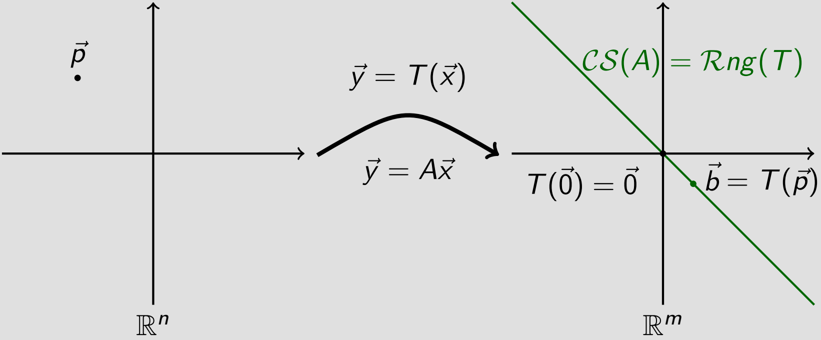

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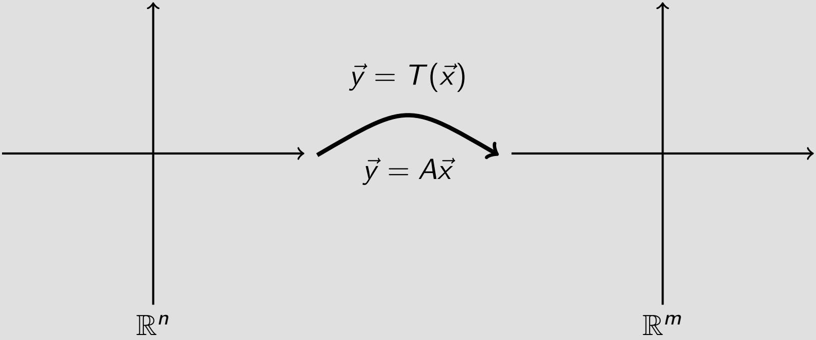

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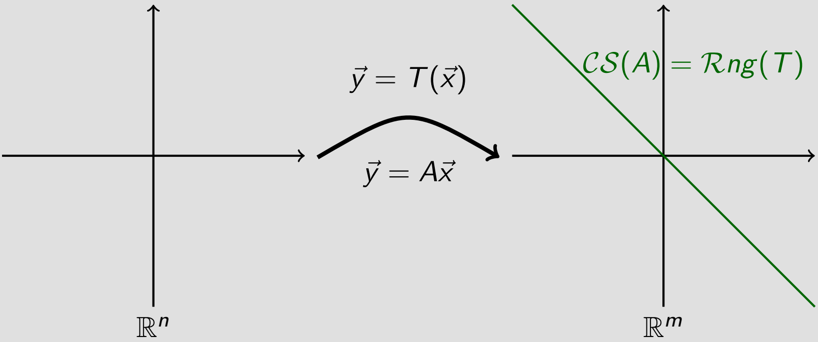

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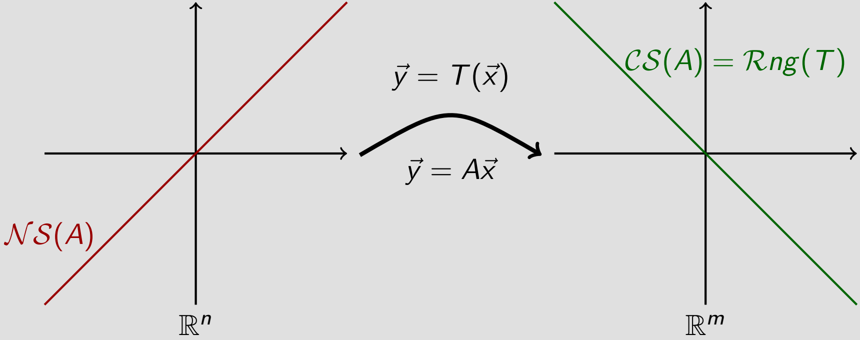

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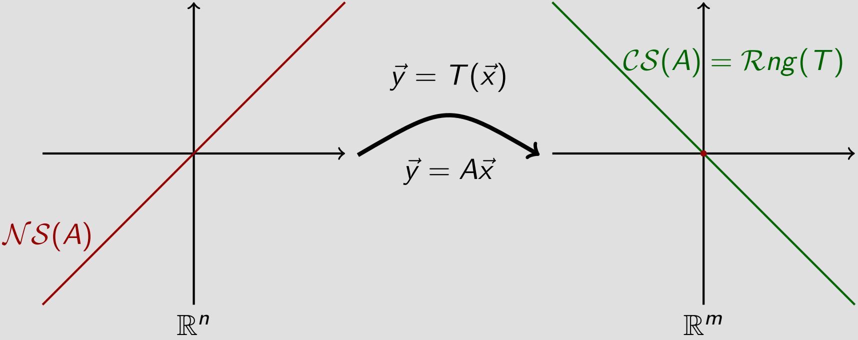

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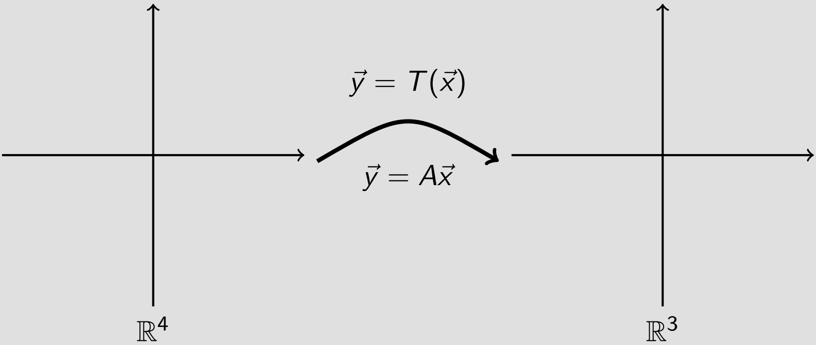

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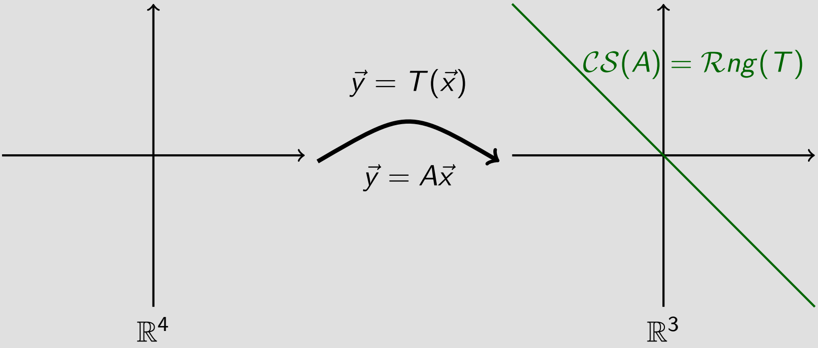

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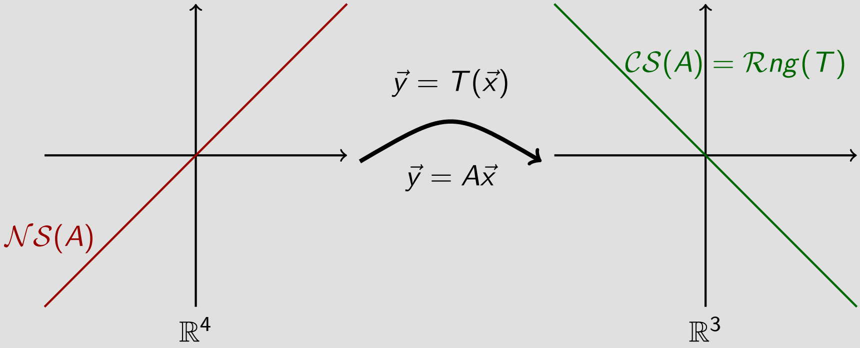

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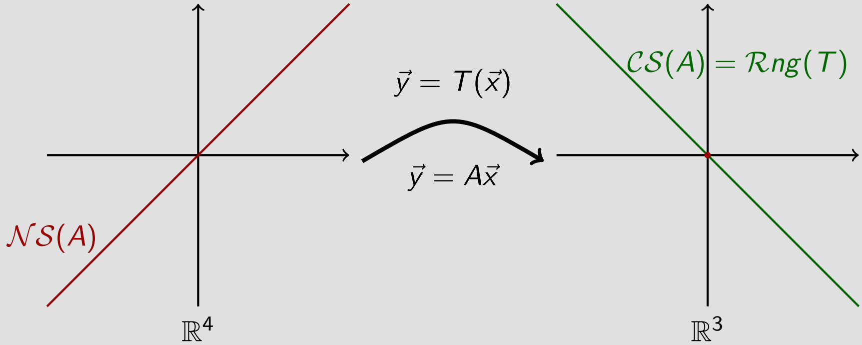

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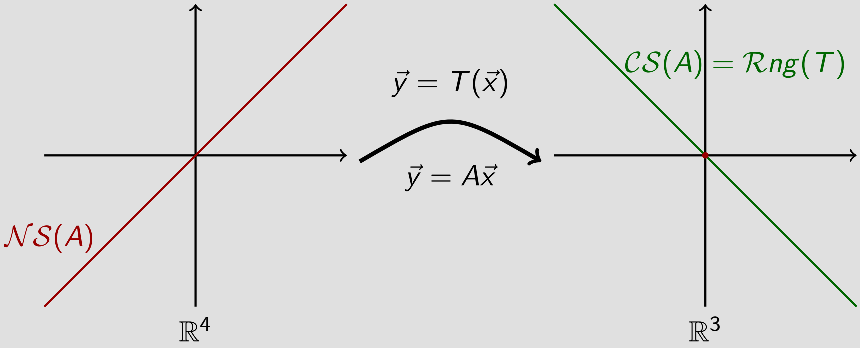

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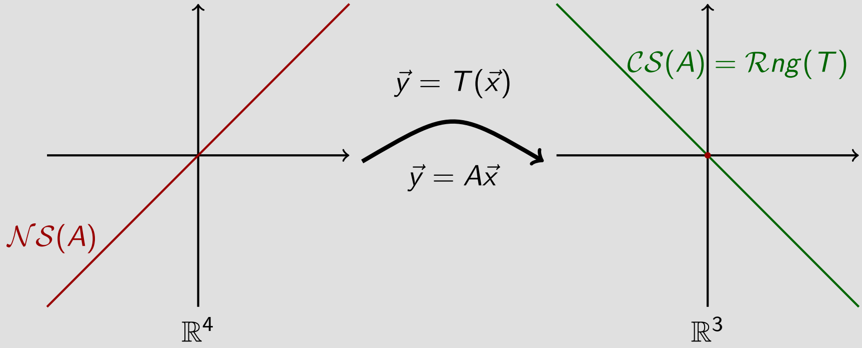

What should we do now? How about row reducing A?

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Basic Fact about Vector SubSpaces

Let $\mathbb{V}$ be a vector subspace. Suppose $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$ are in $\mathbb{V}$.
Then $\mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$ lies in $\mathbb{V}$.