The Inverse of a Matrix

Linear Algebra
MATH 2076

Invertible Matrices

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with $\boxed{A\,C = I = C\,A}$, where I is $n \times n$ identity matrix

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Remember, not all matrices have an inverse.

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So, above is an example of an A with $A^{-1} = A$, and $A \neq \pm I$.

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So, above is another example of an A with $A^{-1} = A$, and $A \neq \pm I$.

The significance of $AC = I$ for a given A

Suppose there is an $n \times n$ matrix C with $AC = I$.

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What does this say about the columns of A?

Finding C (i.e., finding A^{-1})

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Looking for C with $AC = I$. Think about a variable $n \times n$ matrix X. Trying to solve $AX = I$.

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First, $I = [\vec{e}_1 \ \vec{e}_2 \ldots \vec{e}_n]$, where $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, \quad \vec{e}_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$

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so $\mathrm{Col}_j(I) = \vec{e}_j = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ where the one 1 appears in the j^{th} row.

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Next, $\mathrm{Col}_j(AX) =$

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$$AX = I$$

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$$AX = I \quad \text{or} \quad \mathrm{Col}_j(AX) = \mathrm{Col}_j(I)$$

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Look at super-sized augmented matrix $\left[A \,\vdots\, I\right]$.

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Look at super-sized augmented matrix $\left[A \vdots I\right]$. Put into *reduced* REF.

Do elem row ops to get $\left[A \vdots I\right] \xrightarrow[\text{reduced REF}]{\text{row reduce to}} \left[E \vdots F\right]$.

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Looking for C with $AC = I$. Think about a variable $n \times n$ matrix X. Trying to solve $AX = I$. Look at the columns of each side.

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Look at super-sized augmented matrix $\left[A \vdots I\right]$. Put into *reduced* REF.

Do elem row ops to get $\left[A \vdots I\right] \xrightarrow[\text{reduced REF}]{\text{row reduce to}} \left[E \vdots F\right]$. Two possibilities:

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Looking for C with $AC = I$. Think about a variable $n \times n$ matrix X. Trying to solve $AX = I$. Look at the columns of each side.

Next, $\text{Col}_j(AX) = A\,\text{Col}_j(X)$, so we're trying to solve

$$AX = I \quad \text{or} \quad \text{Col}_j(AX) = \text{Col}_j(I) \quad \text{or} \quad A\vec{x} = \vec{e}_j$$

where $\vec{x} = \text{Col}_j(X)$, and we solve for each $1 \le j \le n$.

Look at super-sized augmented matrix $\left[A \,\vdots\, I\right]$. Put into *reduced* REF.

Do elem row ops to get $\left[A \,\vdots\, I\right] \xrightarrow[\text{reduced REF}]{\text{row reduce to}} \left[E \,\vdots\, F\right]$. Two possibilities:

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Example

Determine if $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ is invertible, and find A^{-1} if it exists.

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$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right].$$

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$$
\left[\begin{array}{cccc|cccc}
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1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}\right]
\xrightarrow{R_4 - R_3}
\left[\begin{array}{cccc|cccc}
1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\
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\end{array}\right]
$$

$$
\begin{array}{c}\xrightarrow{R_3 - R_2} \\ \xrightarrow{-R_4}\end{array}
\left[\begin{array}{cccc|cccc}
1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\
1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & -1 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & -1
\end{array}\right]
$$

$$\left[\begin{array}{cccc|cccc}1&2&3&4&1&0&0&0\\1&1&2&3&0&1&0&0\\1&1&1&2&0&0&1&0\\1&1&1&1&0&0&0&1\end{array}\right] \xrightarrow{R_4-R_3} \left[\begin{array}{cccc|cccc}1&2&3&4&1&0&0&0\\1&1&2&3&0&1&0&0\\1&1&1&2&0&0&1&0\\0&0&0&-1&0&0&-1&1\end{array}\right]$$

$$\xrightarrow{R_3-R_2} \xrightarrow{-R_4} \left[\begin{array}{cccc|cccc}1&2&3&4&1&0&0&0\\1&1&2&3&0&1&0&0\\0&0&-1&-1&0&-1&1&0\\0&0&0&1&0&0&1&-1\end{array}\right] \xrightarrow[-R_3]{R_3+R_4} \left[\begin{array}{cccc|cccc}1&2&3&4&1&0&0&0\\1&1&2&3&0&1&0&0\\0&0&1&0&0&1&-2&1\\0&0&0&1&0&0&1&-1\end{array}\right]$$

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So, $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ is invertible, and $A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$.

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This is not hard to do, right?