Matrix Arithmetic

Linear Algebra MATH 2076



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Suppose A is the
$$m \times n$$
 matrix $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}$

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Alternatively, we write $[A]_{ij} = a_{ij}$ to indicate that a_{ij} is the entry of A in the i^{th} row and j^{th} column; more briefly, a_{ij} is the i,j entry of A.

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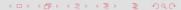
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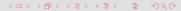


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That is, if $A = [a_{ij}]$ and $B = [b_{ij}]$, then $A + B = [a_{ij} + b_{ij}]$.



Section 2.1

Calculate

$$2\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 5 & 5 \\ -3 & 0 & 2 \end{bmatrix}$$



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Recall that when A is an $m \times n$ matrix and \vec{x} a vector in \mathbb{R}^n ,

$$A\vec{x} = x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_n\vec{a}_n.$$

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Suppose \mathbf{a} is the row vector (i.e., a $1 \times n$ matrix)

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$$

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This agrees with our definition of $A\vec{x}$, right?

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Rows and Columns of a Matrix

Let
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 be an $m \times n$ matrix, say $A = \begin{bmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{m1} & & a_{mj} & \dots & a_{mn} \end{bmatrix}$.

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The j^{th} column of A is the vector

$$\mathsf{Col}_j(A) = \left| egin{array}{c} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{array} \right| \; ; \quad \mathsf{here} \; 1 \leq j \leq n.$$

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The i^{th} column of A is the vector

$$\mathsf{Col}_j(A) = egin{bmatrix} a_{1j} \ a_{2j} \ dots \ a_{mj} \end{bmatrix}$$
; here $1 \leq j \leq n$.

The i^{th} row of A is the row vector

$$Row_i(A) = \begin{bmatrix} a_{i1} & a_{i2} & \dots & a_{in} \end{bmatrix}$$
; here $1 \le i \le m$.

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The i, j entry of AB is simply $[AB]_{ij} = Row_i(A) Col_j(B)$.

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Note that if $A = [a_{ij}]$ and $B = [b_{ij}]$ (careful!), then

$$[AB]_{ij} = \mathsf{Row}_i(A) \; \mathsf{Col}_j(B) = \begin{bmatrix} a_{i1} & a_{i2} & \dots & a_{in} \end{bmatrix} \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{bmatrix} = \sum_{k=1}^n a_{ik} b_{kj}.$$

Notice that

$$AB = [A \operatorname{Col}_1(B) A \operatorname{Col}_2(B) \dots A \operatorname{Col}_n(B)].$$

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Checking that these two formulas are valid is a good exercise to see how well you understand matrix products!

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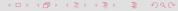
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What is the 0 matrix anyway?



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Is there a multiplicative identity for matrix arithmetic?

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Clearly, for any scalar s and any zero matrix $\mathbf{0}$, $s\mathbf{0} = \mathbf{0}$. Also, for any matrix A, $A + \mathbf{0} = A$ (provided A and $\mathbf{0}$ are of the same dimensions!). Thus $\mathbf{0}$ is the *additive identity* for matrix arithmetic.

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A square matrix is one that has the same number of rows and columns. Note that AB and BA are both defined if and only if A and B are both square matrices with exactly the same size.

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These I's are the multiplicative identities for matrix arithmetic. If A is any square matrix, can we find another square matrix C so that AC = I?