Linear Independence

Linear Algebra
MATH 2076

Linear Combinations

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Suppose $s_1, s_2, \dots, s_p$ are scalars and $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$ are vectors (all in the same space $\mathbb{R}^n$).

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$$s_1\vec{v}_1 + s_2\vec{v}_2 + \cdots + s_p\vec{v}_p$$

a *linear combination* of the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$.

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The *span* of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$,

$$\mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_p\}.$$

is the set of *all* LCs of $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_p$.

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What "is" the span of 7 vectors?

The span of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$

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It depends. . . .

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When is a single vector $\vec{v}$ LD ? (good quiz question!)

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When is a pair of vectors $\vec{v}_1, \vec{v}_2$ LD ?

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Vectors that are *not* LD are said to be *linearly independent*. Linearly independent vectors carry NO redundant information.

Example—three vectors in $\mathbb{R}^3$

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Are the vectors $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\3\end{bmatrix}, \begin{bmatrix}3\\1\\4\end{bmatrix}$ LD or LI?

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But what if we had five (or fifteen) vectors?
Let $A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix}$ be the matrix with columns given by $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

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Easy to see that $\vec{v}_3 = 3\vec{v}_1 - \vec{v}_2$, so $3\vec{v}_1 - \vec{v}_2 - \vec{v}_3 = \vec{0}$ and therefore $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are LD. Also, $\mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \mathcal{S}pan\{\vec{v}_1, \vec{v}_2\}$ which is a 2-plane in $\mathbb{R}^4$.

But what if we had five (or fifteen) vectors?
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