Linear Independence

Linear Algebra
MATH 2076

Linear Combinations and Span

Suppose $s_1, s_2, \dots, s_p$ are scalars and $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$ are vectors (all in the same space $\mathbb{R}^n$).

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$$s_1\vec{v}_1 + s_2\vec{v}_2 + \cdots + s_p\vec{v}_p$$

a *linear combination* of the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$.

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$$0 \cdot \vec{v}_1 + 0 \cdot \vec{v}_2 + \cdots + 0 \cdot \vec{v}_p = \vec{0}.$$

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Recall that the *span* of $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$,

$$\mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \dots \vec{v}_p\}.$$

is the set of *all* LCs of $\vec{v}_1, \vec{v}_2, \dots \vec{v}_p$.

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The span of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$

$$\mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\} = \left\{\vec{w} \mid \vec{w} = s_1\vec{v}_1 + s_2\vec{v}_2 + \cdots + s_p\vec{v}_p\,, s_j \text{ scalars}\right\}$$

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The *span* of a single vector is a line.

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The *span* of a two vectors is:

- a line if the two vectors are parallel

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The *span* of a two vectors is:

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What "is" the span of 7 vectors?

The span of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$

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The *span* of a single vector is a line. Except when this is not true. $\ddot\smile$

The *span* of a two vectors is:

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- a plane if the two vectors are not parallel
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What "is" the span of 7 vectors?

It depends. . . .

Linear Dependence

The vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$ are *linearly dependent* if there is a *non-trivial* LC of them that equals $\vec{0}$:

Linear Dependence

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and (at least) one of the scalars is *non-zero*.

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Linearly dependent vectors carry redundant information.

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and (at least) one of the scalars is *non-zero*.

When is a single vector $\vec{v}$ LD ? (good quiz question!)

Linear Dependence

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When is a pair of vectors $\vec{v}_1, \vec{v}_2$ LD ? (better quiz question!)

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and (at least) one of the scalars is *non-zero*.

Vectors that are *not* LD are said to be *linearly independent*. Linearly independent vectors carry NO redundant information.

Example—three vectors in $\mathbb{R}^3$

Are the vectors $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\3\end{bmatrix}, \begin{bmatrix}3\\1\\4\end{bmatrix}$ LD or LI?

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Thus $\begin{bmatrix}1\\0\\1\end{bmatrix} + \begin{bmatrix}2\\1\\3\end{bmatrix} - \begin{bmatrix}3\\1\\4\end{bmatrix} = \vec{0}$.

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This means that

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which is a plane in $\mathbb{R}^3$.

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Are the vectors $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}2\\2\\7\end{bmatrix}$ LD or LI?

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Are the vectors $\begin{bmatrix}1\\1\\1\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\end{bmatrix}$, $\begin{bmatrix}2\\2\\7\end{bmatrix}$ LD or LI? Well, $2\begin{bmatrix}1\\1\\1\end{bmatrix} + 5\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}2\\2\\7\end{bmatrix}$,

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Are the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 2 \\ -1 \\ 6 \\ 1 \end{bmatrix}$ LD or LI?

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Easy to see that $\vec{v}_3 = 3\vec{v}_1 - \vec{v}_2$, so $3\vec{v}_1 - \vec{v}_2 - \vec{v}_3 = \vec{0}$ and therefore $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are LD.

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Converse true too!

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