# Vectors, Equations, Linear Combinations

Linear Algebra MATH 2076



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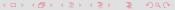
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We write  $\mathbb{R}^{m \times n}$  for the space of all  $m \times n$  matrices.





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It is useful to adopt a notation that helps distinguish between numbers (aka scalars), matrices, and vectors; I'll use arrows. Thus a vector of "size" n (i.e., with n coordinates) is written as

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$$\vec{x} + \vec{y} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + yn \end{bmatrix}.$$

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$$s\vec{x} = s \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} s & x_1 \\ s & x_2 \\ \vdots \\ s & x_n \end{bmatrix}.$$



Suppose  $s_1, s_2, \ldots, s_p$  are scalars and  $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$  are vectors (all in the same space  $\mathbb{R}^n$ ).

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$$s_1\vec{v}_1 + s_2\vec{v}_2 + \cdots + s_p\vec{v}_p$$

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$$\vec{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
 ,  $\vec{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  ,  $\vec{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ .

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Thus every vector in  $\mathbb{R}^3$  can be expressed as a LC of  $\vec{e_1}$ ,  $\vec{e_2}$ ,  $\vec{e_3}$ .

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If A is the coefficient matrix for some SLE, and  $\vec{a_j}$  is the  $j^{\rm th}$  column of A, then

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Thus, an SLE has a solution if and only if ....

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 and  $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ .

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$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}.$$

The product  $A\vec{x}$  is defined to be the LC

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where  $\vec{a_j}$  is the  $j^{\text{th}}$  column of A.

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If  $\vec{b}$  is a vector in  $\mathbb{R}^m$ , we call  $A\vec{x} = \vec{b}$  a matrix equation; here  $\vec{x}$  is the variable.





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which in turn has exactly the same solutions as the matrix equation

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