Vectors, Equations, Linear Combinations

Linear Algebra MATH 2076



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$$\vec{x} + \vec{y} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + yn \end{bmatrix}.$$

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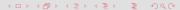
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Thus every vector in \mathbb{R}^3 can be expressed as a LC of $\vec{e_1}$, $\vec{e_2}$, $\vec{e_3}$.

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If A is the coefficient matrix for some SLE, and $\vec{a_j}$ is the $j^{\rm th}$ column of A, then

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Thus, an SLE has a solution if and only if

Linear Algebra

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$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}.$$

The product $A\vec{x}$ is defined to be the LC

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If \vec{b} is a vector in \mathbb{R}^m , we call $A\vec{x} = \vec{b}$ a matrix equation; here \vec{x} is the variable.



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which in turn has exactly the same solutions as the matrix equation

$$A\vec{x} = \vec{b}$$
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