Row Echelon Form
Section 1.2
Exercise # 23

Linear Algebra
MATH 2076

UNIVERSITY OF Cincinnati

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$$A = \left[\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ \vec{a}_4 \ \vec{a}_5 \right]$$

where $\vec{a}_j$ (the j^{th} column of A) is a vector in

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where $\vec{a}_j$ (the j^{th} column of A) is a vector in $\mathbb{R}^3$ (all the coefficients of x_j).

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To say that A has three pivot columns means that any REF for A has three row leaders. So, let E be a REF for A. Then

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To say that A has three pivot columns means that any REF for A has three row leaders. So, let E be a REF for A. Then E must have three non-zero rows, and the row leaders are in 3 of the 5 columns of E, and *every* row of E has a row leader (so E has no zero rows).

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Consistency of the original SLE depends on a REF for the *augmented* coefficient matrix—i.e., whether or not it has a row leader in its last column.

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Thus, $B = \left[\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \vec{a}_4\ \vec{a}_5 \mid \vec{b}\right]$ where $\vec{b}$ is a vector $\mathbb{R}^3$ whose coordinates are the rhs constants from the SLE.

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To row reduce B, we perform **exactly** the same elementary row operations done to obtain E from A.

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To row reduce B, we perform **exactly** the same elementary row operations done to obtain E from A. These elementary row operations transform B into some matrix F,

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To row reduce B, we perform **exactly** the same elementary row operations done to obtain E from A. These elementary row operations transform B into some matrix F, the first 5 columns of F are exactly those of E,

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To row reduce B, we perform **exactly** the same elementary row operations done to obtain E from A. These elementary row operations transform B into some matrix F, the first 5 columns of F are exactly those of E, and so F is in REF (why?).

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This means that F has three row leaders, and these occur in 3 of its first 5 columns.

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Does the SLE have a unique solution, or infinitely many solutions?