Matrix Transformations
Introduction to Linear Transformations

Applied Linear Algebra
MATH 5112/6012

UNIVERSITY OF Cincinnati

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We write $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$ to indicate that T is a transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$, meaning that the input variable $\vec{x}$ comes from $\mathbb{R}^n$ and the resulting output $\vec{y} = T(\vec{x})$ is a vector in $\mathbb{R}^m$.

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An important question is to know which vectors $\vec{b}$ are in the range of T. That is: Given $\vec{b}$, when can we find an $\vec{x}$ in $\mathbb{R}^n$ with $T(\vec{x}) = \vec{b}$?

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Look at: Given $\vec{b}$, when can we find an $\vec{x}$ in $\mathbb{R}^n$ with $T(\vec{x}) = \vec{b}$? Here we are just asking whether or not we can solve $A\vec{x} = \vec{b}$.

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The *range* of T is exactly all rhs vectors $\vec{b}$ such that $A\vec{x} = \vec{b}$ has a solution.

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- $\mathcal{R}ng(T)$ is all $\vec{b}$ in $\mathbb{R}^m$ such that $A\vec{x} = \vec{b}$ has a solution (is consistent)

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There are many equivalent ways to view the range of T!

- $\mathcal{R}ng(T)$ is all images $T(\vec{x})$ for $\vec{x}$ in $\mathbb{R}^n$
- $\mathcal{R}ng(T)$ is all $\vec{b}$ in $\mathbb{R}^m$ such that $\vec{b} = T(\vec{x})$ for some $\vec{x}$ in $\mathbb{R}^n$
- $\mathcal{R}ng(T)$ is all $\vec{b}$ in $\mathbb{R}^m$ such that $A\vec{x} = \vec{b}$ has a solution (is consistent)
- $\mathcal{R}ng(T)$ is the *span* of the columns of A