Subspaces of Euclidean Space $\mathbb{R}^n$

Applied Linear Algebra
MATH 5112/6012

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A simple non-example:

- $\mathbb{V} = \left\{\text{all } \vec{v} \text{ in } \mathbb{R}^4 \text{ with third coordinate -7}\right\}$ is not a subspace

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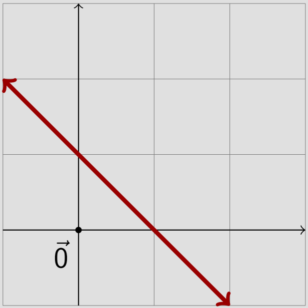

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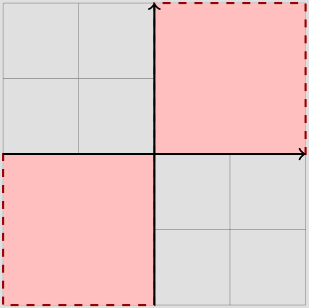

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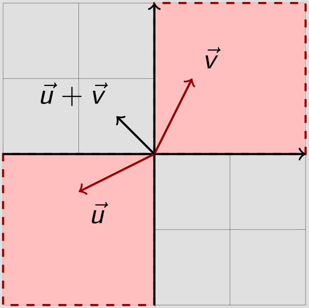

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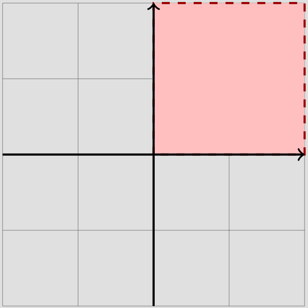

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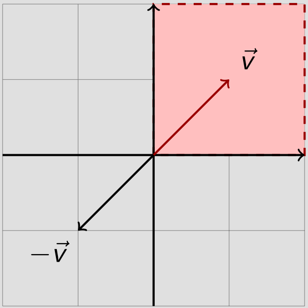

Figure: $x \geq 0$ and $y \geq 0$

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$$\vec{u} + \vec{v} = (s_1 + t_1)\vec{v}_1 + (s_2 + t_2)\vec{v}_2 + \cdots + (s_p + t_p)\vec{v}_p$$

which is a vector in $\mathbb{V}$.

Vector Subspaces—Basic Example

Recall that a collection $\mathbb{V}$ of vectors (in $\mathbb{R}^n$) is a *vector subspace* (of $\mathbb{R}^n$) if and only if

- $\vec{0}$ is in $\mathbb{V}$,
- $\mathbb{V}$ *closed with respect to vector addition* ($\vec{u}, \vec{v}$ in $\mathbb{V} \implies \vec{u} + \vec{v}$ in $\mathbb{V}$)
- $\mathbb{V}$ *closed with respect to scalar mult* (s scalar, $\vec{v}$ in $\mathbb{V} \implies s\vec{v}$ in $\mathbb{V}$)

Let $\mathbb{V} = \mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$. Let's show that $\mathbb{V}$ is closed wrt vector addition. Let $\vec{u}, \vec{v}$ be vectors in $\mathbb{V}$. This means there are scalars $s_1, s_2, \ldots, s_p$ and $t_1, t_2, \ldots, t_p$ with

$$\vec{u} = s_1\vec{v}_1 + \cdots + s_p\vec{v}_p \quad \text{and} \quad \vec{v} = t_1\vec{v}_1 + \cdots + t_p\vec{v}_p$$

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Homework: Show that $\mathbb{V}$ is closed wrt scalar multiplication.

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Just saw that any $\mathbb{V} = \mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p\}$ is both closed wrt vector addition and closed wrt scalar multiplication.

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For any $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$ in $\mathbb{R}^n$, $\mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p\}$ is a vector subspace.

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If $A = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \ldots & \vec{a}_n \end{bmatrix}$, then $\mathcal{CS}(A) = \mathcal{S}pan\{\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n\}$.

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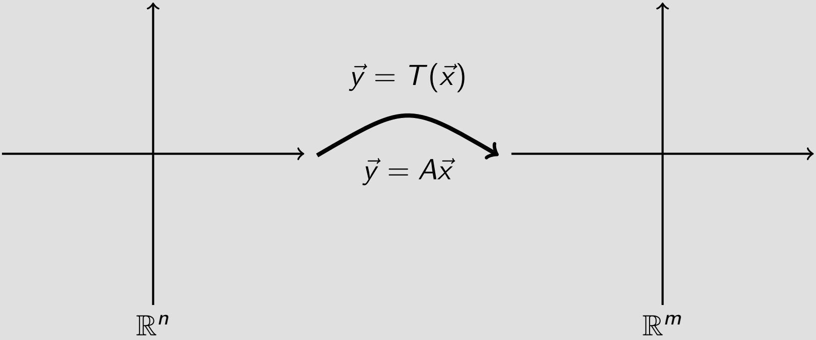

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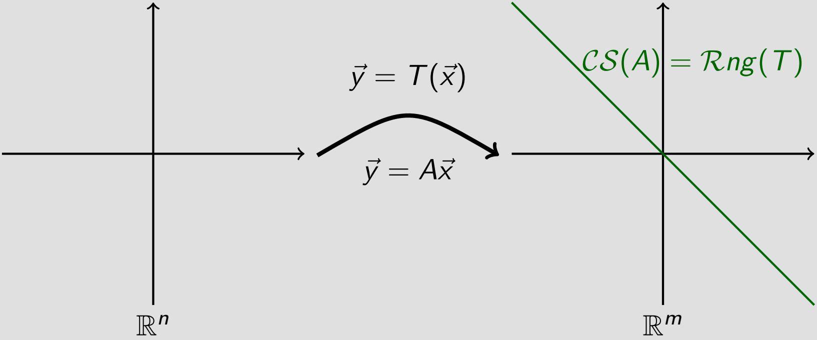

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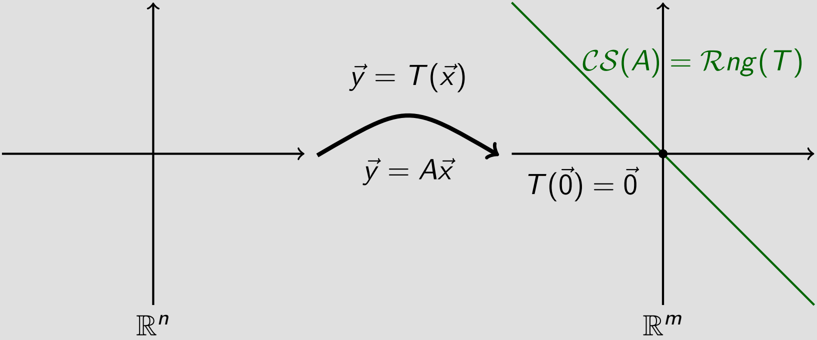

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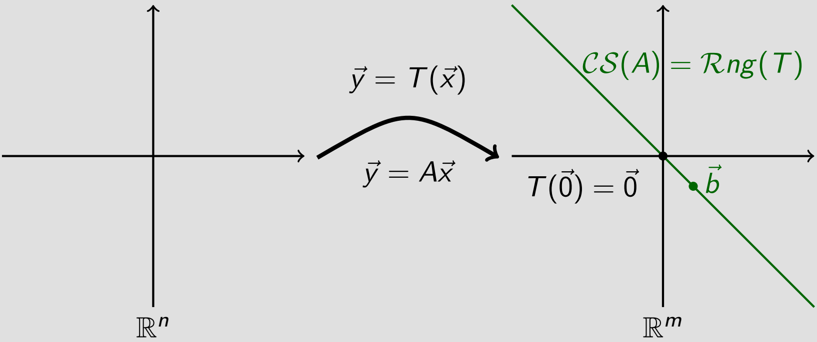

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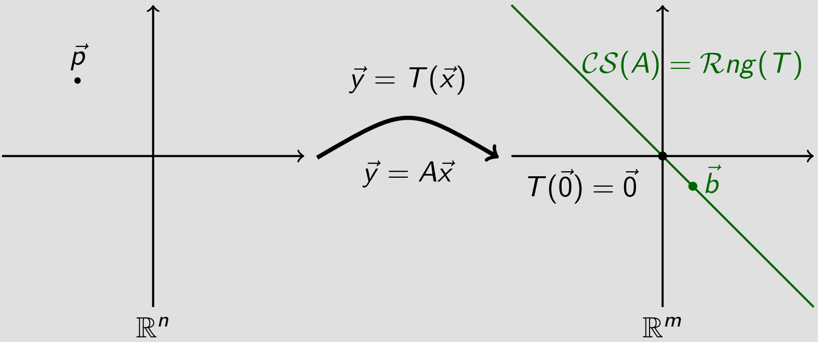

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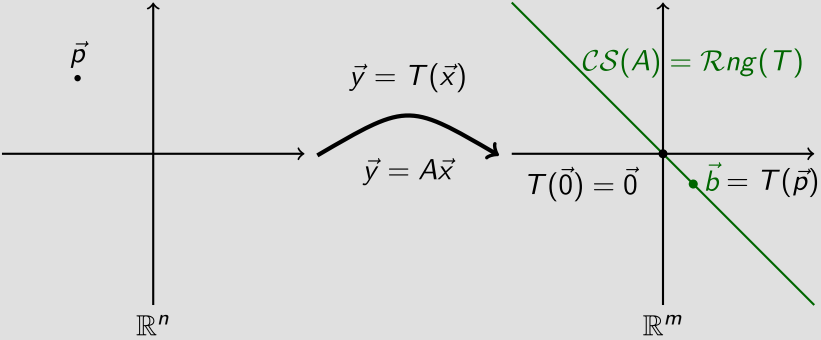

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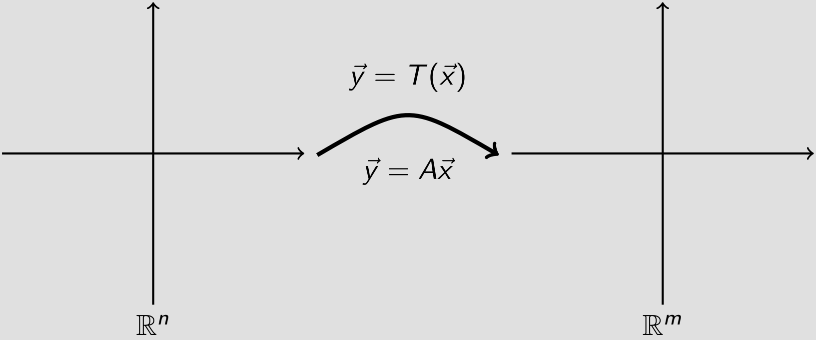

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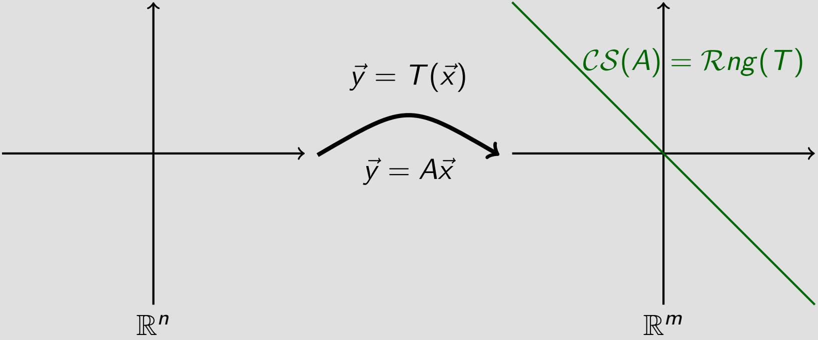

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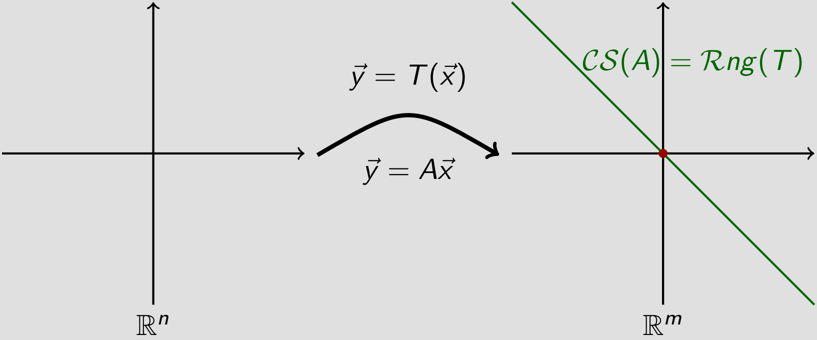

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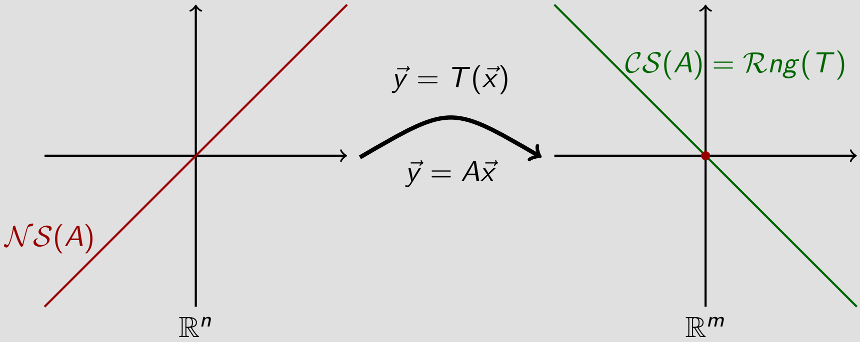

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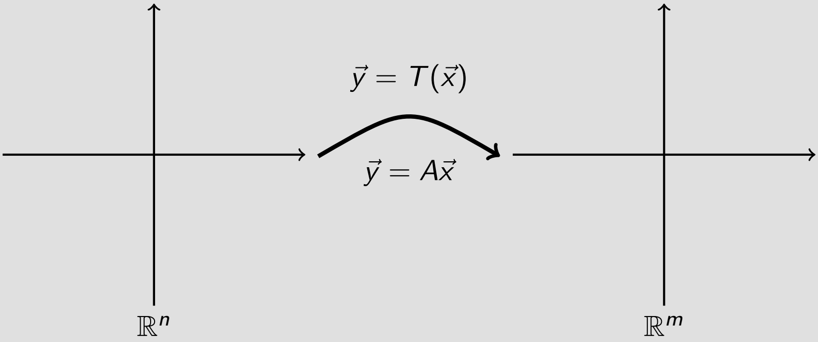

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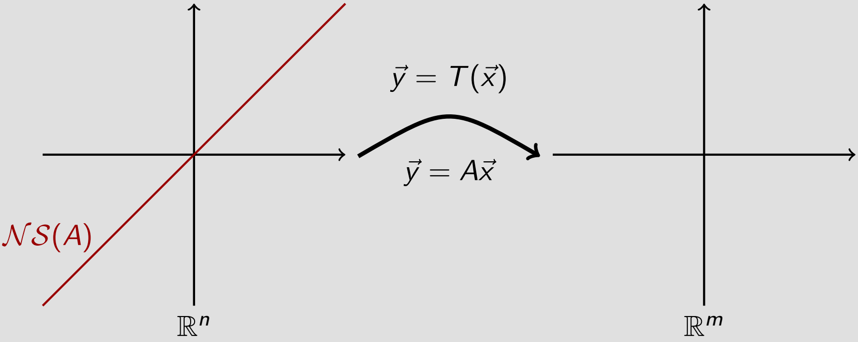

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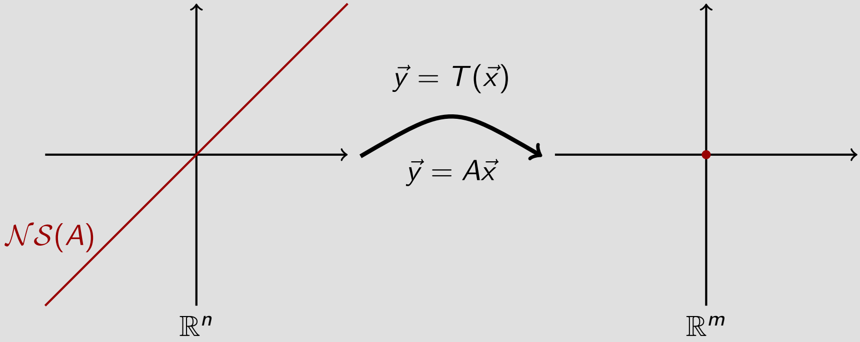

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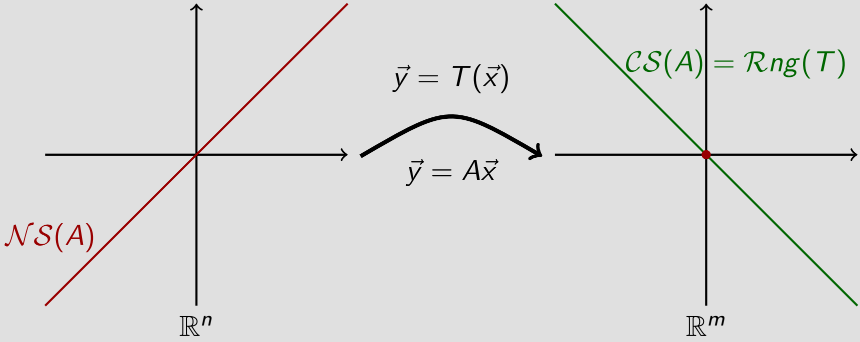

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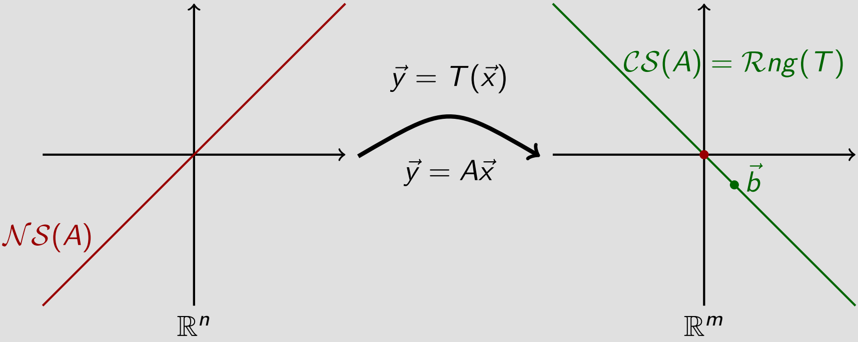

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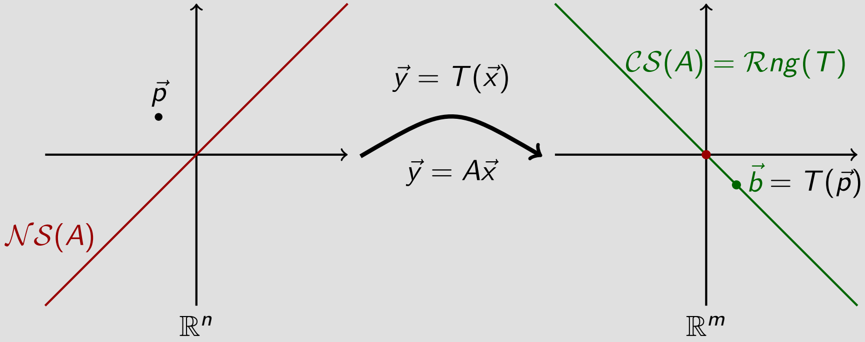

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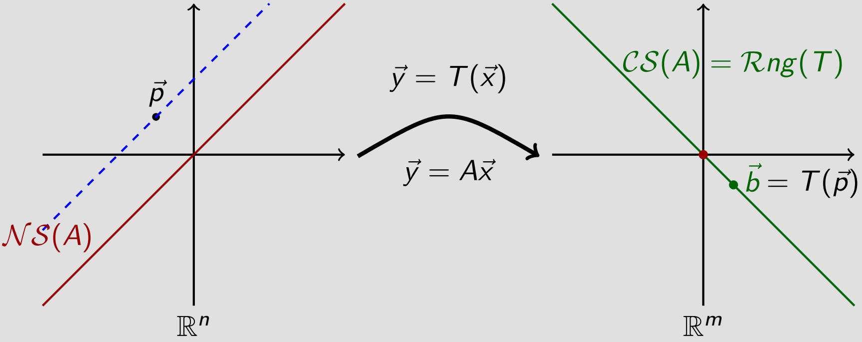

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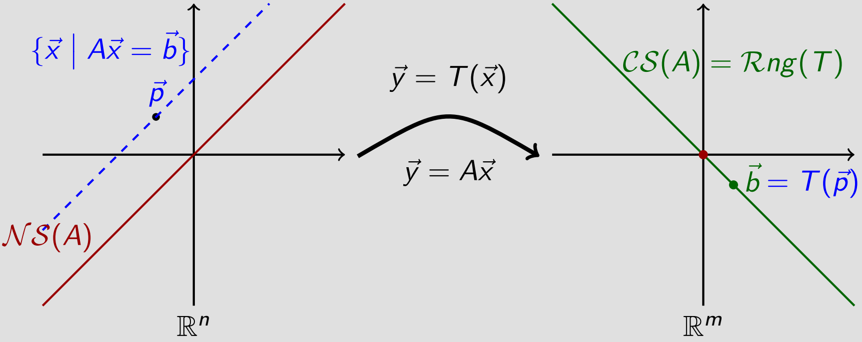

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Find the null space and column space of

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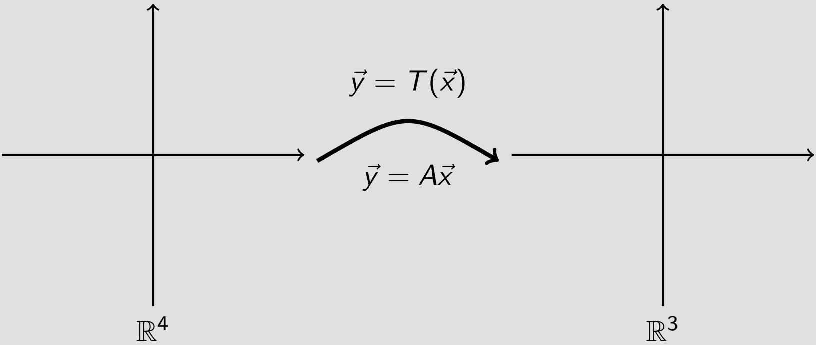

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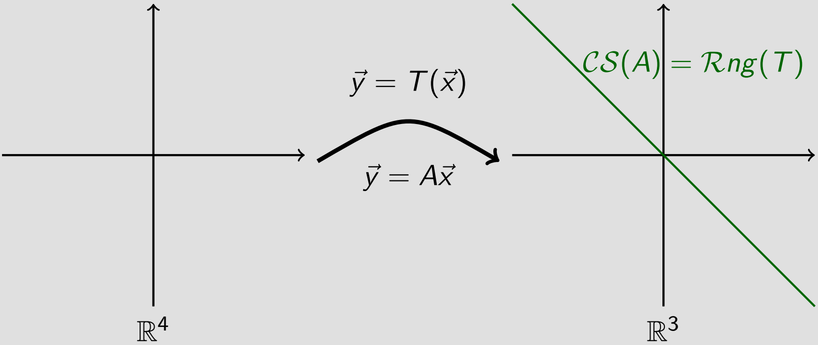

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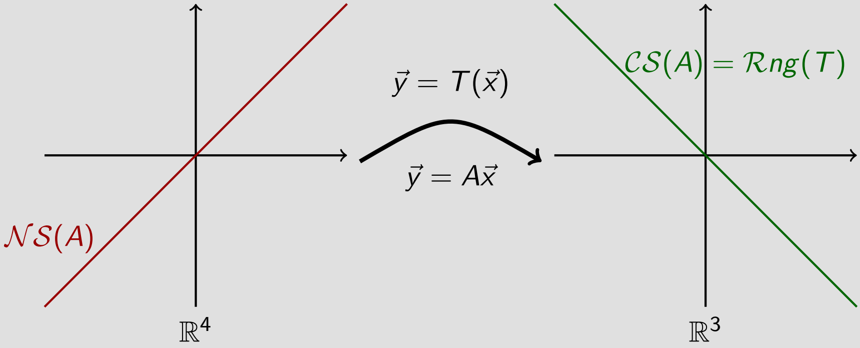

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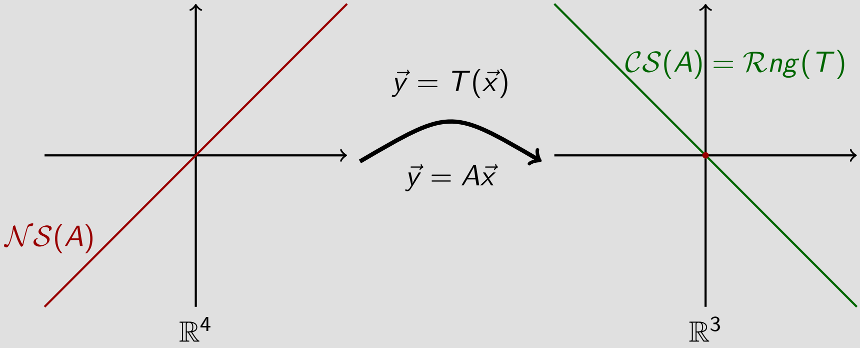

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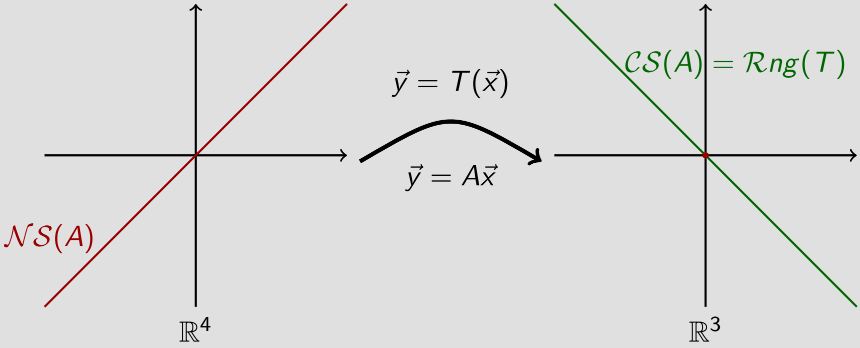

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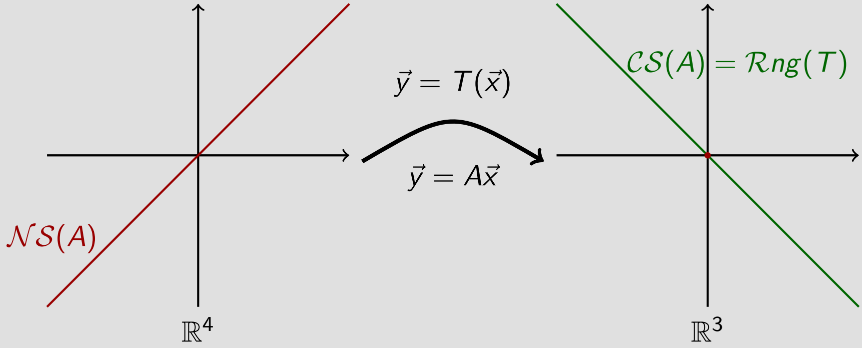

What should we do now? How about row reducing A?

Vector Subspaces—Basic Fact

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Basic Fact about Vector SubSpaces

Let $\mathbb{V}$ be a vector subspace. Suppose $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$ are in $\mathbb{V}$.
Then $\mathcal{S}pan\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p\}$ lies in $\mathbb{V}$.