The Inverse of a Matrix

Applied Linear Algebra Math 5112/6012



Ann	LLOOD.	IDOOK	- ^	~~~	b × b
ADD		LILEAL		PE	
· • • •				_	

A B > A B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA, where I is $n \times n$ identity matrix



An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA, where I is $n \times n$ identity matrix



Some matrices are not invertible:

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA, where I is $n \times n$ identity matrix

Some matrices are not invertible:

$$I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

is not invertible.

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA, where I is $n \times n$ identity matrix

 $I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}.$ Some matrices are not invertible: $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ is not invertible.

How can we tell when a matrix is invertible?

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA, where I is $n \times n$ identity matrix

Some matrices are not invertible: $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ is not invertible.

How can we tell when a matrix is invertible?

How can we find such a matrix C?

 $I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & & 1 \end{bmatrix}.$

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

$$C_1 =$$

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

$$C_1 = C_1 I$$

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

$$C_1 = C_1 I = C_1 (A C_2)$$

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

$$C_1 = C_1 I = C_1 (A C_2) = (C_1 A) C_2$$

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

$$C_1 = C_1 I = C_1 (A C_2) = (C_1 A) C_2 = I C_2$$

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

$$C_1 = C_1 I = C_1 (A C_2) = (C_1 A) C_2 = I C_2 = C_2$$

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

Suppose we have $C_1A = I = AC_2$. Then

$$C_1 = C_1 I = C_1 (A C_2) = (C_1 A) C_2 = I C_2 = C_2$$

so $C_1 = C_2$.

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

Suppose we have $C_1A = I = AC_2$. Then

$$C_1 = C_1 I = C_1 (A C_2) = (C_1 A) C_2 = I C_2 = C_2$$

so $C_1 = C_2$.

We call C the *inverse* of A and write $A^{-1} = C$.

(4 部) (4 国) (4 国)

An $n \times n$ matrix A is *invertible* if and only if there is another $n \times n$ matrix C with AC = I = CA.

When this holds, there is only ONE such matrix C.

Suppose we have $C_1A = I = AC_2$. Then

$$C_1 = C_1 I = C_1 (A C_2) = (C_1 A) C_2 = I C_2 = C_2$$

so $C_1 = C_2$.

We call C the *inverse* of A and write $A^{-1} = C$.

Remember, not all matrices have an inverse.

Remember, for numbers $x^{-1} = x \implies x^2 = 1$, so $x = \pm 1$.

イロン 不聞き 不良さ 不良という

Remember, for numbers $x^{-1} = x \implies x^2 = 1$, so $x = \pm 1$.

Look at

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

イロト イロト イヨト イヨト 三日

Remember, for numbers $x^{-1} = x \implies x^2 = 1$, so $x = \pm 1$.

Look at

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

イロト イロト イヨト イヨト 三日

Remember, for numbers $x^{-1} = x \implies x^2 = 1$, so $x = \pm 1$.

Look at

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

イロン 不聞き 不良さ 不良という

Remember, for numbers $x^{-1} = x \implies x^2 = 1$, so $x = \pm 1$.

Look at

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

SO,

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{-1} =$$

3

Remember, for numbers $x^{-1} = x \implies x^2 = 1$, so $x = \pm 1$.

Look at

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

SO,

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

3

イロト イヨト イヨト

Remember, for numbers $x^{-1} = x \implies x^2 = 1$, so $x = \pm 1$.

Look at

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

SO,

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

So, above is an example of an A with $A^{-1} = A$, and $A \neq \pm I$.

(日) (同) (三) (三)

Look at

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

臣

Look at

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

臣

Look at

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

臣

Look at

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

so,

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^{-1} =$$

Applied Linear Algebra

臣

Look at

SO,

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}.$$

Applied Linear Algebra

臣

Look at

SO,

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}.$$

So, above is another example of an A with $A^{-1} = A$, and $A \neq \pm I$.

(日) (同) (三) (三)

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

$$A\vec{x} = AC\vec{b}$$

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

$$A\vec{x} = AC\vec{b} = I\vec{b}$$

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

$$A\vec{x} = AC\vec{b} = I\vec{b} = \vec{b}.$$
Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Try $\vec{x} = C\vec{b}$. Get $A\vec{x} = AC\vec{b} = I\vec{b} = \vec{b}$.

When there is C with AC = I, we know that for any rhs \vec{b} , $A\vec{x} = \vec{b}$ has a solution,

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Try $\vec{x} = C\vec{b}$. Get $A\vec{x} = AC\vec{b} = I\vec{b} = \vec{b}$.

When there is C with AC = I, we know that for any rhs \vec{b} , $A\vec{x} = \vec{b}$ has a solution, namely $\vec{x} = C\vec{b}$.

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Try $\vec{x} = C\vec{b}$. Get $A\vec{x} = AC\vec{b} = I\vec{b} = \vec{b}$.

When there is C with AC = I, we know that for any rhs \vec{b} , $A\vec{x} = \vec{b}$ has a solution, namely $\vec{x} = C\vec{b}$. Solutions always exist for any rhs \vec{b} !

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Try $\vec{x} = C\vec{b}$. Get $A\vec{x} = AC\vec{b} = I\vec{b} = \vec{b}$.

When there is C with AC = I, we know that for any rhs \vec{b} , $A\vec{x} = \vec{b}$ has a solution, namely $\vec{x} = C\vec{b}$. Solutions always exist for any rhs \vec{b} !

What does this say about a REF for A?

Suppose there is an $n \times n$ matrix C with AC = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Try $\vec{x} = C\vec{b}$. Get $A\vec{x} = AC\vec{b} = I\vec{b} = \vec{b}$.

When there is C with AC = I, we know that for any rhs \vec{b} , $A\vec{x} = \vec{b}$ has a solution, namely $\vec{x} = C\vec{b}$. Solutions always exist for any rhs \vec{b} !

What does this say about a REF for A?

What does this say about the columns of A?

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose that \vec{p} is some solution, so $A\vec{p} = \vec{b}$.

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

$$CA\vec{p} = C\vec{b}.$$

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

$$I\vec{p} = CA\vec{p} = C\vec{b}.$$

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

$$\vec{p} = I\vec{p} = CA\vec{p} = C\vec{b}.$$

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose that \vec{p} is some solution, so $A\vec{p} = \vec{b}$. Multiply both sides by C to get

$$\vec{p} = I\vec{p} = CA\vec{p} = C\vec{b}.$$

When there is C with CA = I, we know that $A\vec{x} = \vec{b}$ has the unique solution, $\vec{x} = C\vec{b}$.

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose that \vec{p} is some solution, so $A\vec{p} = \vec{b}$. Multiply both sides by C to get

$$\vec{p} = I\vec{p} = CA\vec{p} = C\vec{b}.$$

When there is C with CA = I, we know that $A\vec{x} = \vec{b}$ has the unique solution, $\vec{x} = C\vec{b}$. If there is a solution (for some rhs \vec{b}), then the solution is unique!

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose that \vec{p} is some solution, so $A\vec{p} = \vec{b}$. Multiply both sides by C to get

$$\vec{p} = I\vec{p} = CA\vec{p} = C\vec{b}.$$

When there is C with CA = I, we know that $A\vec{x} = \vec{b}$ has the unique solution, $\vec{x} = C\vec{b}$. If there is a solution (for some rhs \vec{b}), then the solution is unique!

What does this say about a REF for A?

Suppose there is an $n \times n$ matrix C with CA = I.

Look at the equation $A\vec{x} = \vec{b}$ for some given rhs \vec{b} .

Suppose that \vec{p} is some solution, so $A\vec{p} = \vec{b}$. Multiply both sides by C to get

$$\vec{p} = I\vec{p} = CA\vec{p} = C\vec{b}.$$

When there is C with CA = I, we know that $A\vec{x} = \vec{b}$ has the unique solution, $\vec{x} = C\vec{b}$. If there is a solution (for some rhs \vec{b}), then the solution is unique!

What does this say about a REF for A?

What does this say about the columns of A?

First, A may be non-invertible, right?

3

First, A may be non-invertible, right? There may be no such C.

First, A may be non-invertible, right? There may be no such C. Looking for C with AC = I.

골 동 김 골 동

< □ > < /□ > <</p>

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I.

< 4 **1 1** ► <

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

First,
$$I = \begin{bmatrix} \vec{e_1} & \vec{e_2} \dots \vec{e_n} \end{bmatrix}$$
, where $\vec{e_1} = \begin{bmatrix} 1\\0\\\vdots\\0 \end{bmatrix}$, $\vec{e_2} = \begin{bmatrix} 0\\1\\\vdots\\0 \end{bmatrix}$, \dots , $\vec{e_n} = \begin{bmatrix} 0\\0\\\vdots\\1 \end{bmatrix}$

< □ > < /□ > <</p>

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

First,
$$I = \begin{bmatrix} \vec{e}_1 & \vec{e}_2 \dots \vec{e}_n \end{bmatrix}$$
, where $\vec{e}_1 = \begin{bmatrix} 1\\0\\\vdots\\0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0\\1\\\vdots\\0 \end{bmatrix}$, ..., $\vec{e}_n = \begin{bmatrix} 0\\0\\\vdots\\1 \end{bmatrix}$
so $\operatorname{Col}_j(I) = \vec{e}_j = \begin{bmatrix} 0\\\vdots\\1\\\vdots\\0 \end{bmatrix}$ where the one 1 appears in the j^{th} row.

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_{j}(AX) =$

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

AX = I

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

AX = I or $Col_j(AX) = Col_j(I)$

(日)

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \text{Col}_j(X)$, and we solve for each $1 \le j \le n$.

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \text{Col}_j(X)$, and we solve for each $1 \le j \le n$. Look at super-sized augmented matrix $\begin{bmatrix} A \\ \vdots \end{bmatrix} I$.

- 4 同 ト 4 国 ト 4 国 ト

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \text{Col}_j(X)$, and we solve for each $1 \le j \le n$. Look at super-sized augmented matrix $[A \vdots I]$. Put into *reduced* REF.

- 4 回 ト 4 ヨ ト 4 ヨ ト

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \operatorname{Col}_{j}(X)$, and we solve for each $1 \leq j \leq n$. Look at super-sized augmented matrix $\begin{bmatrix} A \\ \vdots \end{bmatrix} I$. Put into *reduced* REF. Do elem row ops to get $\begin{bmatrix} A \\ \vdots \end{bmatrix} I \xrightarrow[reduced REF]{reduced REF} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$.

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \operatorname{Col}_j(X)$, and we solve for each $1 \le j \le n$. Look at super-sized augmented matrix $\begin{bmatrix} A \\ \vdots \end{bmatrix} I$. Put into *reduced* REF. Do elem row ops to get $\begin{bmatrix} A \\ \vdots \end{bmatrix} \xrightarrow[reduced REF]{reduced REF} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$. Two possibilities:

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_j(AX) = A \operatorname{Col}_j(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \operatorname{Col}_j(X)$, and we solve for each $1 \le j \le n$. Look at super-sized augmented matrix $\begin{bmatrix} A \\ \vdots I \end{bmatrix}$. Put into *reduced* REF. Do elem row ops to get $\begin{bmatrix} A \\ \vdots I \end{bmatrix} \xrightarrow{\text{row reduce to}}_{\text{reduced REF}} \begin{bmatrix} E \\ \vdots F \end{bmatrix}$. Two possibilities: If $E \ne I$.

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_{j}(AX) = A \operatorname{Col}_{j}(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \operatorname{Col}_j(X)$, and we solve for each $1 \le j \le n$. Look at super-sized augmented matrix $\begin{bmatrix} A & \\ I \end{bmatrix}$. Put into *reduced* REF. Do elem row ops to get $\begin{bmatrix} A & \\ I \end{bmatrix} \xrightarrow{\text{row reduce to}}_{\text{reduced REF}} \begin{bmatrix} E & F \end{bmatrix}$. Two possibilities:

If $E \neq I$,

If E = I,

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_{j}(AX) = A \operatorname{Col}_{j}(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \operatorname{Col}_j(X)$, and we solve for each $1 \le j \le n$. Look at super-sized augmented matrix $\begin{bmatrix} A \\ \vdots I \end{bmatrix}$. Put into *reduced* REF. Do elem row ops to get $\begin{bmatrix} A \\ \vdots I \end{bmatrix} \xrightarrow[reduced REF]{reduced ref} \begin{bmatrix} E \\ \vdots F \end{bmatrix}$. Two possibilities:

If $E \neq I$, then A not invertible.

If E = I,

First, A may be non-invertible, right? There may be no such C.

Looking for C with AC = I. Think about a variable $n \times n$ matrix X. Trying to solve AX = I. Look at the columns of each side.

Next, $\operatorname{Col}_{j}(AX) = A \operatorname{Col}_{j}(X)$, so we're trying to solve

$$AX = I$$
 or $\operatorname{Col}_j(AX) = \operatorname{Col}_j(I)$ or $A\vec{x} = \vec{e_j}$

where $\vec{x} = \operatorname{Col}_j(X)$, and we solve for each $1 \le j \le n$. Look at super-sized augmented matrix $\begin{bmatrix} A \\ \vdots I \end{bmatrix}$. Put into *reduced* REF. Do elem row ops to get $\begin{bmatrix} A \\ \vdots I \end{bmatrix} \xrightarrow[reduced REF]{reduced REF} \begin{bmatrix} E \\ \vdots F \end{bmatrix}$. Two possibilities:

If $E \neq I$, then A not invertible.

If
$$E = I$$
, then $F = A^{-1}$.

Look at super-sized augmented matrix $\begin{bmatrix} A \\ \vdots \end{bmatrix}$.

3
< □ > < /□ > <</p>

Look at super-sized augmented matrix $[A \\ \vdots I]$. Put into *reduced* REF.

Do elementary row operations to get
$$\begin{bmatrix} A \\ \vdots \end{bmatrix} \xrightarrow[\text{reduced to}]{\text{reduced REF}} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$$
.

Do elementary row operations to get
$$\begin{bmatrix} A \\ \vdots \end{bmatrix} \xrightarrow[\text{rew reduce to}]{\text{reduced REF}} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$$
.

Get two possibilities:

Do elementary row operations to get $\begin{bmatrix} A \\ \vdots \end{bmatrix} \xrightarrow[\text{reduced to}]{\text{reduced REF}} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$.

Get two possibilities:

If $E \neq I$,

Do elementary row operations to get
$$\begin{bmatrix} A \\ \vdots \end{bmatrix} \xrightarrow[\text{reduced to}]{\text{reduced REF}} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$$
.

Get two possibilities:

If $E \neq I$, If E = I,

(日)

Do elementary row operations to get $\begin{bmatrix} A \\ \vdots \end{bmatrix} \xrightarrow[\text{reduced to}]{\text{reduced REF}} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$.

Get two possibilities:

If $E \neq I$, then A not invertible. If E = I,

Do elementary row operations to get $\begin{bmatrix} A \\ \vdots \end{bmatrix} \xrightarrow{\text{row reduce to}}_{\text{reduced REF}} \begin{bmatrix} E \\ \vdots \end{bmatrix} F$.

Get two possibilities:

If $E \neq I$, then A not invertible.

If E = I, then $F = A^{-1}$.

Determine if
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
 is invertible, and find A^{-1} if it exists.

<ロ> <四> <四> <四> <四> <四> <四</p>

Determine if
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
 is invertible, and find A^{-1} if it exists.

Gotta row reduce

$$\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

臣

イロト イヨト イヨト イヨト

 $\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

▲ロト ▲圖ト ▲国ト ▲国ト 三国 - の々で

 $\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & | & 0 & 0 & -1 & 1 \end{bmatrix}$

▲ロト ▲園ト ▲画ト ▲画ト 三直 めんの

$$\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & | & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & | & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 - R_2} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & | & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \end{bmatrix}$$

 $\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & | & 0 & 0 & -1 & 1 \end{bmatrix}$ $\underbrace{ \begin{smallmatrix} R_3 - R_2 \\ \hline -R_4 \\ \hline -R_4 \\ \hline \end{smallmatrix}} \begin{pmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \\ \end{bmatrix} \underbrace{ \begin{smallmatrix} R_3 + R_4 \\ \hline -R_3 \\ \hline -R_3 \\ \hline \end{smallmatrix}} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 \\ \end{bmatrix}$ 0 0 1 -1

(日) (四) (三) (三) (三)

12

$$\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & | & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & | & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & | & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 & | & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 1 \\ \hline R_3 - R_2 \\ \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ \hline R_3 - R_4 \\ \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & | \\ 0 & 0 & 1 & 0 & | & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 \\ \hline R_2 - R_1 \\ \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & | \\ 0 & 0 & 1 & 0 & | & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & | & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & | & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 & | & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & | & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 & | & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_3 + R_4} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & | & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_2 + R_3 + R_4} \xrightarrow{-R_2} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_2 + R_3 + R_4} \xrightarrow{-R_2}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 2 & | & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & | & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & | & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_3 + R_4} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_3 + R_4} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_2 + R_3 + R_4} \xrightarrow{R_2 + R_3 + R_4} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 & 3 & 4 & | & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \xrightarrow{R_1 - 2R_2} \xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & | & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & | & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \xrightarrow{R_2 - 2R_1} \xrightarrow{R_2 - 2R_1} \xrightarrow{R_2 - 2R_1} \xrightarrow{R_2 - 2R_1} \xrightarrow{R_2 - 2R_2} \xrightarrow{R_2 - 2R_1} \xrightarrow{R_2 - 2R_$$

So,
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
 is invertible, and $A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

So,
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
 is invertible, and $A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Should check that

臣

イロト イヨト イヨト イヨト

So,
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
 is invertible, and $A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Should check that

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} = I$$

(日) (四) (E) (E) (E) (E)

So,
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
 is invertible, and $A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Should check that

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} = I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(日) (四) (E) (E) (E) (E)

So,
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
 is invertible, and $A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Should check that

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} = I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is not hard to do, right?

<ロト < 回 > < 回 > 、 < 回 >

э