Matrix Arithmetic

Applied Linear Algebra
MATH 5112/6012

Matrix Notation

Suppose A is the $m \times n$ matrix $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}$.

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Alternatively, we write $[A]_{ij} = a_{ij}$ to indicate that a_{ij} is the entry of A in the i^{th} row and j^{th} column; more briefly, a_{ij} is the i, j entry of A.

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Example

Calculate

$$2\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & 1\end{bmatrix}-\begin{bmatrix}2 & 5 & 5\\ -3 & 0 & 2\end{bmatrix}$$

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$$2\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & 1\end{bmatrix} - \begin{bmatrix}2 & 5 & 5\\ -3 & 0 & 2\end{bmatrix} = \begin{bmatrix}2 & 4 & 6\\ 0 & -2 & 2\end{bmatrix} + \begin{bmatrix}-2 & -5 & -5\\ 3 & 0 & -2\end{bmatrix}$$

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If A is square and $A^{\mathsf{T}} = A$, then A is called a *symmetric* matrix.

Matrix Multiplication

Recall that when A is an $m \times n$ matrix and $\vec{x}$ a vector in $\mathbb{R}^n$,

$$A\vec{x} = x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_n\vec{a}_n.$$

where $\vec{a}_j$ is the j^{th} column of A.

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Row Vector Times a Column Vector

Suppose **a** is the row vector (i.e., a $1 \times n$ matrix)

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$$

and $\vec{b}$ is the column vector (i.e., an $n \times 1$ matrix) $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$.

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This agrees with our definition of $A\vec{x}$, right?

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Note that if $A = [a_{ij}]$ and $B = [b_{ij}]$ (careful!), then

$$[AB]_{ij} = \text{Row}_i(A)\ \text{Col}_j(B) = \begin{bmatrix} a_{i1} & a_{i2} & \dots & a_{in} \end{bmatrix} \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{bmatrix} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

Some Useful Formulas

Notice that

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Checking that these formulas are valid is a good exercise to see how well you understand matrix products!

Examples

Calculate

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 1 \end{bmatrix} \tag{1}$$

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(3)

(4)

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Matrix Multiplication Anomalies

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What is the $\mathbf{0}$ matrix anyway?

Special Matrices

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A *square* matrix is one that has the same number of rows and columns. Note that AB and BA are both defined if and only if A and B are both square matrices with exactly the same size.

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These I's are the *multiplicative identities* for matrix arithmetic. If A is any square matrix, can we find another square matrix C so that $AC = I$?