

LINEAR ALGEBRA Exam 1 Review

I Systems of Equations

- (1) General System of Linear of Equations
 - What is an SLE?
 - What constitutes a solution?
 - What are the: coefficients? variables? constants?
 - What are corresponding *vector* and *matrix* equations?
- (2) Geometric Viewpoint—lines, planes in \mathbb{R}^3 and k -planes in \mathbb{R}^n
- (3) Solution Trichotomy—none, a unique one, infinitely many
- (4) Finding Solutions to $A\vec{x} = \vec{b}$
 - manipulation phase into upper triangular system
 - row reduction via elementary row operations to REF
 - back-substitution to get solutions, or use reduced REF
 - Gaussian elimination (GE) and Gauss-Jordan elimination (GJE)
 - what is general solution? how to find?
- (5) Row Equivalent Systems
 - why do these have the same solutions?
- (6) Homogeneous Systems $A\vec{x} = \vec{0}$
 - why is there always a solution?
 - what is general solution? how to find?
 - what is connection to non-homogeneous system?
 - what is connection with *linear independence*?
- (7) Existence and Uniqueness of Solutions to $A\vec{x} = \vec{b}$
 - when do solutions exist? when unique?
 - connection with *linear combinations*?
 - connection with *linear independence*?
 - how to find all \vec{b} so consistent?

II Matrices

- (1) Notation
 - entries, columns, rows
- (2) Arithmetic
 - scalar multiplication and addition
 - matrix multiplication
 - inverses (what? how get? “big theorem”)
- (3) Connection with Linear Systems of Equations
 - coefficient and augmented matrices
 - elementary row operations
 - row equivalent matrices
 - GE and row echelon form (REF)
 - GJE and reduced row echelon form (RREF)
- (4) Interpretation of (R)REF: Existence & Uniqueness of Solutions
 - the BIG theorem
 - zero rows (consistency—existence of solutions)
 - number of columns w/o row leader (“free” variables—uniqueness of solutions)
 - number of row leaders (what?)
- (5) Invertible (or Non-Singular) Matrices
 - definition/meaning; connection with inverses
 - how to calculate?
 - connection with solutions to SLEs
 - basic properties
 - the BIG theorem
- (6) Transpose
 - definition/meaning
 - symmetric and skew-symmetric matrices
 - upper and lower triangular matrices
 - diagonal matrices