COMMUNICATING MATHEMATICS, OUR PROBLEM SESSION, AND WRITING UP HOMEWORK SOLUTIONS

In one form or another, every mathematical communication proceeds as follows. First, a statement is made to the effect that: "This... is true." Then a *Proof* is provided. The actual content of the proof varies depending on the circumstances, which include, among other things: the speaker, the audience, the situation. Here are some possibilities.

- A researcher talking to another in her field.
- A researcher talking to another who is in a different field.
- A researcher presenting at a professional meeting.
- A teacher talking to a student (in class, or in her office).
- Students talking (at a blackboard, or at a bar:-).
- A paper written for a professional journal.
- A note written for the local newspaper.
- A student writing a HW solution.

Please note that I expect a lot out of your HW solutions! (See below.)

Here are things I would like to see when you present a solution at our Problem Session.

(1) Start by writing down what is given and/or what we know. Use pictures!
(2) Write the word *Proof* and then write exactly what it is that must be shown.
(3) Explain, in words, the general idea and the basic facts behind 'why it works'.
(4) Finally, present your proof; be sure to draw lotsa pictures.

Our Problem Session is meant to be an informal gathering where you present solutions to *each other*. There should be interaction between the presenter and the audience; e.g., questions. Do not worry about writing sentences, and feel free to use mathematical notation (e.g., $\exists$, $\forall$, $x \in X$) and abbreviations, and diagrams too. Remember that you are trying to explain your ideas to your fellow students.

Please note that I expect *everybody* to attend and participate in the Problem Session. If you do not, please, instead, turn in carefully written solutions to the 'assigned' exercises.

On the back, and also in a separate handout, I provide some suggestions that I would like you to follow when you write up your HomeWork Solutions.

Date: September 22, 2010.

Here are a few thoughts to keep in mind when you are working on and writing up your **HomeWork Solutions**. Please also read the additional handout.

(1) First and foremost, feel free to work together, but turn in your own work. I welcome questions and tend to be generous with my hints, but please do not come to my office the day a problem is due and expect help on it.

(2) Please turn in a neat stapled stack of papers.

(3) What you actually hand in, your final finished version, should be as polished as you can make it. This probably means that you will have previously written up at least sketchy solutions. Please expect to do a fair amount of *rewriting*. Please do not hand in work with parts crossed out; either use a pencil and erase or rewrite. I would guess that you should rewrite each solution *several* times.

(4) Please write using complete sentences that form paragraphs and so forth. I find it best to use short simple sentences; avoid long complicated sentences.

(5) Please minimize the use of special mathematical notation. For example, the symbols $\forall$, $\exists$, $\therefore$, etc. are not appropriate in the middle of a sentence. These are suitable in 'displayed' information. And you should display all the important 'stuff'.

(6) Do use commonly accepted notation (e.g., for functions, sets, etc.) and never invent new notation when there is already some available.

(7) Please provide a statement clearly indicating precisely what it is that you are about to prove. You can, if you want, label your statement as a <u>Theorem</u> or <u>Claim</u> or whatever. Write the word <u>Proof</u>, and then give your proof. Throughout your proof, constantly tell the reader (me!) exactly what it is that you are about to demonstrate. Be sure to indicate the end of your proof. (I like to use a symbol such as $\square$.)

(8) Throughout your proof, constantly tell the reader (me!) exactly what it is that you are about to demonstrate.

(9) Your proof should combine "logic", your hypotheses, and possibly other mathematical facts (e.g., theorems that we have proved in class) into an argument that establishes the asserted conclusion. Do *not* use a 'proof by contradiction' when you are actually doing a contrapositive argument. (The contrapositive of "$P \implies Q$" is "not $Q \implies$ not P"). In a true 'proof by contradiction' of "$P \implies Q$", one starts with the hypotheses "P and not Q" and then one deduces a contradiction such as 1=0.

(10) Please be extra careful about the order in which you use your quantifiers. Many many mathematicians (myself included) are careless and we write such things as "...$a \in A_n$ for all $n \in \mathbb{N}$", but this is really nonsense.

(11) In your arguments you can make free use of anything that we have proven in class, and of course all logic rules and basic axioms and definitions. Anything else that you use should be proved.

I ask that each of you read some literature on writing mathematics. Sometime (by the end of October) I will ask that your turn in a brief 'report' discussing what you have read. This will be the first of two **Written Assignments**. I will have more to say about this at a later time.