Tips on Writing in Mathematics

Notes. An important purpose of courses like Math 244 (Elementary Real Analysis) and Math 252 (Abstract Algebra) is to learn to write proofs and other more or less formal mathematical "paragraphs."

The style and language of mathematical writing may not seem especially natural at first sight. But—with practice and experience—one soon finds that adhering carefully to mathematical writing conventions actually simplifies and streamlines the process of writing proofs, and it helps assure that your ideas are intelligible to others.

Here, then, are some useful tips for writing proofs and proof-like prose in mathematics:

1. *Write in complete "sentences."* The quotes are there because mathematical sentences in mathematics may include not just ordinary words but also symbols, equations, inequalities, etc. For instance, it's fine to write

   Because $x > 3$ we know $x^2 > 9$ and $x^3 > 27$.

   On the other hand, it's usually *not* good form to write

   $x > 3$, $x^2 > 9$, $x^3 > 27$.

   One problem with the latter is that there's no "connective tissue": The reader can't tell whether you're listing your favorite inequalities, asserting that something implies something else, or what.

2. *Use punctuation, capital letters, and other standard grammatical niceties.* These devices are used in ordinary English to help the reader see clearly what's being said or implied. Consider, for instance, the different possible meanings of this notice supposedly posted near an Australian beach:

   crocodiles don't swim here

   Would you swim here or not? Is the sign intended for human or reptile readers?

   In practice, many mathematical sentences convey complex ideas, and so naturally have correspondingly complex structures. It's especially important, therefore, to write mathematics in as clear and unambiguous a manner as possible, and to give the reader every possible aid in deciphering meaning.

3. *Be sure your sentences "scan."* Proofs and solutions must be not only correct but also intelligible to a reader. An excellent way to assure the latter is to read each sentence back to yourself. (Doing this silently rather than aloud may reduce ridicule from neighbors.) A sentence with proper English grammar and syntax may be mathematically right or wrong. (Every mathematician has seen eloquently expressed proofs that boil down to nonsense.) But a sentence without these attributes is almost surely wrong or, worse, meaningless to a reader.

4. *Be clear.* That's easily said but, admittedly, not easily done: complicated or difficult ideas are naturally hard to express clearly. But the work is worth doing, and practice is essential. Giving you practice with and feedback on your mathematical writing is an important part of courses like ERA and Abstract Algebra.

5. *Use standard mathematical symbols and notations, and use them in standard ways.* For instance, the notations $(1, 3)$, $[1, 3]$, and $\{1, 3\}$ all have precise but different meanings—one is an open interval, one is a closed interval, and one is a set with just two members. Violating these conventions, arbitrary as they may be, is asking for trouble. It is far from clear, for instance, what such notations as

$$[x : x^2 > 2] \quad \text{and} \quad \mathbb{Q} : (-\sqrt{2}, \sqrt{2})$$

really mean. By contrast, the expressions

$$\{x : x^2 > 2\} \quad \text{and} \quad \{x \in \mathbb{Q} : x^2 > 2\}$$

are clear and unambiguous.

6. *Take care with word order.* Because mathematical language conveys complex thoughts, word order can be crucial. Consider these two statements:

> For every $\epsilon > 0$ there exists an integer n such that $0 < 1/n < \epsilon$.
> There exists an integer n such that $0 < 1/n < \epsilon$ for every $\epsilon > 0$.

The first statement is true; it's a version of what's sometimes called the Archimedean principle for real numbers. The second statement is false.

7. *Just say no to "it."* The harmless-looking word "it" commits countless crimes in mathematics. For example, the connection between stationary points of f and roots of f' has sometimes been put as vaguely as this:

> It is maximum or minimum when it's flat, and that happens when it's a zero of its derivative.

That's just too many pronouns, and it's far from clear what each one refers to. Just say no:

> A function f may have a maximum or minimum value where the graph of f is flat. This can occur at a value of x which is a root of the derivative function f'.

8. *Make it look easy.* A musician planning a recital invests hours of practice and study, and hits plenty of false notes. The recital itself, however, skips all the practice and study (and most of the false notes). In the same way, a finished mathematical proof should be the polished result, rather than the basic process, of whatever informal thinking, experiments, and false starts may have contributed along the way. While it's sometimes okay to include some record of the investigative phase of proving a result, it's important *not* to confuse such material with the proof itself. The best proofs are clear, concise, and couched in standard mathematical language.

9. *Don't say too much—or too little.* Respect, but don't overtax, your reader's intelligence and willingness to work. Ideas in your proof should be clear and accessible to your reader—someone with your own level of intelligence and knowledge.