Common Errors in Writing Mathematics

The following things are often confused:

$$\text{equations,} \qquad \text{expressions,} \qquad \text{functions.}$$

An **equation** is a statement that two things are equal, like

$$x^3 + 3x - 2 = 0 \qquad \text{or} \qquad y = x^2.$$

An **expression** is some mathematical symbolism without a verb in it, for instance

$$x + 1 \qquad \text{or} \qquad (x+1)(y-2) \qquad \text{or} \qquad \frac{f(x+a) - f(x)}{a}.$$

By "verb" I don't necessarily mean something written as a word; more likely it will be an equal sign, or an inequality sign.

A **function** is a process that takes inputs and makes outputs. The symbol f denotes a function, but $f(3)$ does not; $f(3)$ is a *result* or *value* of a function. So, do not refer to the "function $f(3)$". You mean "the value $f(3)$" or "the output $f(3)$".

Exception. You may refer to "the function $f(x)$" instead of "the value $f(x)$". This usage is technically incorrect, but accepted. The point is, x is a dummy variable here. You are merely saying that f is a function because it applies to inputs, rather than referring to the specific output when the specific letter x is input. Similarly, if z is your dummy variable, you may refer to "the function $g(z)$" instead of "the value $g(z)$". However, in both cases it would be better to say "the function f" or "the function g". Or, leave out "function" and just say "$f(x)$".

Examples. Consider

A function is an equation. For instance, $f(x) = x^2 - 3$.

No! A function is a process. A function is *defined* by giving an equation that provides a *formula* in terms of a dummy variable, but the equation is not itself the function.

Example 2. What is wrong with:

Let $f(x) = x^2$. In

$$\hat{f}(x) = f(2x-1) - f(3x+2) \tag{1}$$

for the first function f you replace the dummy variable by $2x - 1$ in the equation of f, getting $(2x - 1)^2$.

Answer: Two things. There is only one function f in (1). It is *evaluated* twice. Second, there is no "equation of f"; rather, there is a definition of f, the statement that starts with "Let".

Correct rewrite: Let $f(x) = x^2$. In

$$\hat{f}(x) = f(2x-1) - f(3x+2)$$

for the first evaluation of f you substitute $2x - 1$ for the dummy variable x in the definition of f, getting $(2x - 1)^2$.

But why use awkward word expressions like "first evaluation of f" when you can use brief symbols:

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Better rewrite: Let $f(x) = x^2$. In

$$\hat{f}(x) = f(2x{-}1) - f(3x{+}2)$$

to evaluate $f(2x{-}1)$ you substitute $2x-1$ for the dummy variable x in the definition of f, getting $(2x-1)^2$.

Example 3. What is wrong with writing

Given the function $x + 10$, evaluate the function for $f(4)$.

Answer: $x + 10$ is not a function; it is an expression. Apparently this expression is meant to be the *output* of a function named f, since the letter f is used later. Also (but minor), there is redundancy later in the sentence.

Corrected: Given the function $f(x) = x + 10$, evaluate $f(4)$.

Even better (because it avoids the exceptional wording "function $f(x)$"):

Given the function f defined by $f(x) = x + 10$, evaluate $f(4)$,

or

Let $f(x) = x + 10$. Evaluate $f(4)$.

This last version avoids the issue of what is rightly called a function by avoiding the word. The notation carries the day. Notice that the best version is also the shortest.

Example 4. What is wrong with writing

Let $f(x) = x^2 - 2x$. To find $f(2/x)$ just plug in $2/x$ everywhere for x and complete the function.

Answer: The difficulty is the phrase "complete the function". The author means: do any obvious simplifications to the *expression* $(2/x)^2 - 2(2/x)$. So "complete the simplification" would be all right, though not as good as the single word "simplify". Because $x^2 - 2x$ is not the function itself, but rather the output, "complete the function" is incorrect.

Example 5. What is wrong with writing

Let $f(x) = 2^x/(x + 1)$. Solve for $f(3)$.

Answer: There is nothing to *solve* for. You are merely *evaluating*. If the task had been

Find x if $f(x) = 3$,

then there would be something to solve for, because you would start with $3 = 2^x/(x + 1)$ and have to isolate x.

Example 6. What's not right here?

Let $g(x) = (x{+}1)(x{+}2)$. To evaluate $g(y{-}1)$ we must plug $y - 1$ into g.

Answer: $y - 1$ was already plugged into g when we wrote $g(y{-}1)$. That already is a name for the output we want. To evaluate it, that is, find a more familiar name for it, we must plug $y - 1$ into the the right side of the **defining equation** $g(x) = (x{+}1)(x{+}2)$, obtaining $y(y{+}1)$.

Final Remark. Please refer to "function notation", not "functional notation". The adjective form of (mathematical) function is "function", not "functional". This is because "functional" already has another meaning in ordinary English. "Functional notation" means any notation that works well, whether or not it has anything to do with functions, or even with math.

For more information, see the handouts from my manuscript *Short Guide to Writing Mathematics*.