Is the Value Premium a Proxy for Time-Varying Investment Opportunities: Some Time-Series Evidence

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Abstract

We uncover a positive stock market risk-return tradeoff after controlling for the covariance of market returns with the value premium. Fama and French (1996) conjecture that the value premium proxies for investment opportunities; therefore, by ignoring it, early specifications suffer from an omitted variable problem that causes a downward bias in the risk-return tradeoff estimation. We also document a positive relation between the value premium and its conditional variance, and the estimated conditional value premium is strongly countercyclical. The latter evidence supports the view that value is riskier than growth in bad times, when the price of risk is high.

I. Introduction

The capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) fails to explain the stock return data along two important dimensions. First, Fama and French (1993), for example, show that the CAPM does not account for the cross-section of stock returns, e.g., the value premium and the size premium.[1] Second, many authors, e.g., Campbell (1987), Glosten, Jagannathan, and Runkle (1993), Whitelaw (1994), and Brandt and Kang (2004), find a weak or negative risk-return tradeoff in the stock market across time, in contrast with the positive relation stipulated by Merton's (1973) intertemporal CAPM (ICAPM).

The CAPM-related anomalies suggest that the stock market might act as a hedge against changes in investment opportunities, as illustrated in Merton's (1973) ICAPM. In particular, Fama and French (1996) argue that the value and size premia move closely with investment opportunities and include these premia as additional risk factors in their three-factor model—one of the most influential and successful empirical asset pricing models. Consistent with Fama and French's conjecture, Liew and Vassalou (2000) find that the value premium forecasts output growth in many developed economies. Campbell and Vuolteenaho (2004), Brennan, Wang, and Xia (2004), Hahn and Lee (2006), and Petkova (2006) show that the value premium is correlated with innovations in their measures of investment opportunities. Gomes, Kogan, and Zhang (2003), Zhang (2005), and Lettau and Wachter (2007) develop equilibrium models to establish a link between the value premium and investment opportunities.

[1] The value premium is the return on a portfolio that is long in stocks with high book-to-market equity ratios (value stocks) and short in stocks with low book-to-market equity ratios (growth stocks). The size premium is the return on a portfolio that is long in stocks with small capitalizations and short in stocks with big capitalizations.

Motivated from Fama's (1991) conjecture of an explicit link between the cross-sectional and time-series stock return predictabilities, we investigate in this paper whether the value premium constructed from the cross-section of stocks sheds light on the on-going debate about the intertemporal relation between stock market risk and return. In particular, if the value premium is a proxy for investment opportunities, Merton's (1973) ICAPM indicates that the conditional excess stock market return, $E_t(R_{t+1})$, is determined by its conditional variance, $\sigma^2_{M,t}$, and its conditional covariance with the value premium, $\sigma_{MH,t}$:

(1) $$E_t(R_{t+1}) = \gamma_M \sigma^2_{M,t} + \gamma_H \sigma_{MH,t}.$$

The parameter γ_M is the coefficient of relative risk aversion and should be positive. The coefficient γ_H is equal to $-\frac{J_{WF}}{J_W}$, where $J(W(t),\ F(t),\ t)$ is the indirect utility function of the representative agent with subscripts denoting partial derivatives, $W(t)$ is wealth, and $F(t)$ is a vector of state variables that describe investment opportunities. Similarly, the conditional value premium, $E_t(HML_{t+1})$, is determined by its conditional variance, $\sigma^2_{H,t}$, and its conditional covariance with the stock market return, $\sigma_{MH,t}$:

(2) $$E_t(HML_{t+1}) = \gamma_M \sigma_{MH,t} + \gamma_H \sigma^2_{H,t}.$$

For robustness, as in French, Schwert, and Stambaugh (1987), we estimate equations (1) and (2) using both the realized variance model advocated by Merton (1980) and the autoregressive conditional heteroskedasticity (ARCH) model advanced by Engle (1982). We obtain similar results using both estimation techniques, and our main findings can be summarized as follows. First, over the modern period 1963 to 2005, there is a weak risk-return relation in the U.S. stock market.[2] However, it becomes significantly positive after we control for the covariance of stock market returns with the value premium; and conditional stock market returns are positively related to the covariance as well. Second, we document a new finding on a significantly positive relation between the value premium and its conditional variance after controlling for its covariance with stock market returns. Also, because of the strongly countercyclical movement in the conditional variance, the conditional value premium tends to be high during business recessions and to be low during business expansions. Lastly, to address the potential concern over data mining, we estimate the ICAPM using Fama and French's (1998) international data, and find similar results for the world market as well as most of the other G7 countries. Overall, our results are consistent with the conjecture that the value premium is a proxy for investment opportunities.

Scruggs (1998) estimates a bivariate GARCH model using the long-term interest rate as a proxy for investment opportunities. Scruggs and Glabadanidis (2003), however, find that the results from that study are somewhat sensitive to the assumption of a constant correlation coefficient between stock market returns and the long-term interest rate. Guo and Whitelaw (2006) use the consumption-wealth ratio proposed by Lettau and Ludvigson (2001a) as a proxy for investment opportunities and find results very similar to ours.[3] Guo and Whitelaw (2006) focus on the stock market risk-return

[2] We mainly focus on the modern period because some recent studies, e.g., Campbell and Vuolteenaho (2004), Ang and Chen (2007), Petkova and Zhang (2005), and Fama and French (2006), find that the CAPM explains the value premium in the early period 1926 to 1962. One possible explanation is that, as suggested by Campbell and Vuolteenaho (2004), the value premium is a poor proxy for changes in investment opportunities in the early period. Similarly, we find that the value premium helps uncover the positive risk-return tradeoff only in the modern period.

[3] One can use Campbell and Shiller's (1988) log-linearization method to show that the scaled stock price, e.g., the consumption-wealth ratio, is a linear function of conditional stock market variance and conditional covariance of stock

tradeoff. In contrast, our main motivation here is to test the hypothesis of whether the value premium proxies for investment opportunities.[4] Moreover, Scruggs (1998) and Guo and Whitelaw (2006) use only U.S. data, while we provide international evidence as well. In a paper circulated after the first version of this paper, Brandt and Wang (2006) use the value premium as a proxy for investment opportunities to investigate whether the stock market risk-return tradeoff changes across time.

Following Fama and French (1993, 1996), many authors have used the value premium as an unconditional risk factor. Jagannathan and Wang (1996), Lettau and Ludvigson (2001b), and Petkova and Zhang (2005) have also investigated the role of conditioning information in explaining the value premium. These authors find that value stocks tend to be much riskier than growth stocks during an economic downturn, when the price of risk is high. However, Lewellen and Nagel (2006) cast doubt on the empirical relevance of the conditional models used in these studies. By contrast, our evidence of strongly countercyclical variation in the expected value premium lends support to the view that the conditioning information is economically important for understanding the value anomaly.

The value premium is an empirically motivated risk factor and has limitations; for example, Ferson and Harvey (1999) find that it does not help explain the dynamics of stock returns. Nevertheless, our evidence raises the bar for some alternative hypotheses by uncovering a close link between time-series and cross-sectional stock return predictability. Such a link is well established in

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market returns with the shock to investment opportunities. Consistent with the hypothesis that the value premium is a proxy for investment opportunities, we find that the predictive power of the value premium for stock market returns is very similar to that of the consumption-wealth ratio.

[4] A few recent studies also uncover a positive risk-return tradeoff by using (1) alternative measures of the conditional stock market variance (Ghysels, Santa-Clara, and Valkanov (2005)); (2) alternative measures of the conditional stock market return (Pastor, Sinha, and Swaminathan (2006)); (3) longer historical stock return data (Lundblad (2006)); and (4) conditioning variables extracted from a large set of macroeconomic variables (Ludvigson and Ng (2007)).

Merton's (1973) ICAPM; however, it poses a challenge to the irrational pricing explanation by Lakonishok, Shleifer, and Vishny (1994) and the data mining explanation by MacKinlay (1995) for the value premium.

The remainder of the paper is organized as follows. We present the estimation results of the realized variance model in Section II and of the bivariate GARCH model in Section III. We provide the international evidence in Section IV and discuss the main findings in Section V. We offer some concluding remarks in Section VI.

II. The Realized Variance Model

A Data Descriptions

We obtain daily and monthly data of the Fama and French three factors from Ken French at Dartmouth College. Daily data are available over the period July 2, 1963, to December 31, 2005, and monthly data are available over the period July 1926 to December 2005. Following Merton (1980) and Andersen, Bollerslev, Diebold, and Labys (2003), among many others, we use the sum of the squared daily returns in a quarter as a measure of realized variance for both stock market returns and the value premium.[5] Realized covariance is measured as the sum of the cross-product of daily excess stock market returns with the daily value premium. We also construct quarterly return data by aggregating monthly returns through simple compounding.

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[5] We focus on quarterly data rather than monthly data because Ghysels, Santa-Clara, and Valkanov (2005) argue that realized variance is a function of long distributed lags of squared past daily returns. Also, as in French, Schwert, and

[Insert Figure 1 Here]

Figure 1 plots realized stock market variance, $v_{M,t}^2$ (dashed line); realized value premium variance, $v_{H,t}^2$ (solid line); and realized covariance between the stock market return and the value premium, $v_{MH,t}$ (thick solid line). The variable $v_{M,t}^2$ rose dramatically during the 1987 stock market crash and reverted to the normal level shortly after. Because many authors, e.g., Schwert (1990), argue that the 1987 crash is unusual in many ways, we follow Campbell, Lettau, Malkiel, and Xu (2001) and replace realized variance for 1987:Q4 with the second-largest observation in the sample. The variable $v_{MH,t}$ is almost always negative, suggesting that the market provides a hedge for changes in investment opportunities, given the premise that they are proxied by the value premium. The absolute value of $v_{MH,t}$ tends to be relatively high just before or during business recessions (dated by the National Bureau of Economic Research (NBER)), as denoted by the shaded areas. The variables $v_{M,t}^2$ and $v_{MH,t}$ usually move in opposite directions. Also, realized variance of the value premium, $v_{H,t}^2$, is negatively related to the covariance, $v_{MH,t}$, while it closely relates to realized variance of the stock market return, $v_{M,t}^2$. As we show in the next subsection, these patterns help explain why ignoring the hedge for changes in investment opportunities leads to a downward bias in the estimate of the risk-return tradeoff in the stock market.

[Insert Table 1 Here]

Table 1 presents summary statistics for the excess stock market return and the value premium as well as their realized variances and covariance over the period 1963:Q3 to 2005:Q4. Panel A shows

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Stambaugh (1987), we find essentially the same results by correcting the serial correlation in daily return data. For brevity, these results are not reported here but are available on request.

that the excess stock market return, R_t, is negatively related to the value premium, HML_t, with a correlation coefficient of –0.46. Also, consistent with Figure 1, the variables $v_{M,t}^2$, $v_{H,t}^2$, and $v_{MH,t}$ are closely related to each other; however, the correlation is far from perfect. Panel B shows that the realized second moments are relatively persistent: The autocorrelation coefficients are 0.53, 0.72, and 0.56 for $v_{M,t}^2$, $v_{H,t}^2$, and $v_{MH,t}$, respectively. Therefore, realized variances and covariance are good predictors of their future levels.

B. Estimation Results of Merton's (1973) ICAPM

We can rewrite equations (1) and (2) in the realized return form and use realized variances and covariance as proxies for the conditional variances and covariance, respectively:

$$\begin{aligned} R_{t+1} &= \alpha_M + \gamma_{MM} v_{M,t}^2 + \gamma_{HM} v_{MH,t} + \varepsilon_{M,t+1} \\ HML_{t+1} &= \alpha_H + \gamma_{MH} v_{MH,t} + \gamma_{HH} v_{H,t}^2 + \varepsilon_{H,t+1} \end{aligned}, \tag{3}$$

where $\varepsilon_{M,t+1}$ and $\varepsilon_{H,t+1}$ are shocks to the market return and the value premium, respectively. Merton's (1973) ICAPM also imposes restrictions on the coefficients in equation (3): $\alpha_M = \alpha_H = 0$, $\gamma_{MM} = \gamma_{MH} = \gamma_M$, and $\gamma_{HM} = \gamma_{HH} = \gamma_H$. We estimate equation (3) using the GMM (generalized methods of moments) by Hansen (1982) and report the estimation results in Table 2.

[Insert Table 2 Here]

Row 1 of panel A in Table 2 replicates the familiar result that realized stock market variance, $v_{M,t}^2$, has weak forecasting power for the excess stock market return, R_{t+1}: Its coefficient is positive but only marginally significant, with an adjusted R-squared of 1.6%. However, it remains positive and becomes significant at the 1% level after we control for realized covariance of stock market returns with the value premium, $v_{MH,t}$ (row 2). Interestingly, the effect of $v_{MH,t}$ is also significantly positive, and the adjusted R-squared increases to 4.8% in row 2 from 1.6% in row 1. Because $v_{M,t}^2$ and $v_{MH,t}$ are negatively correlated (as shown in Figure 1), our results suggest that the specification in row 1 of panel A suffers from a classic omitted variable problem, which leads to a downward bias in the estimate of the risk-return tradeoff.[6]

Row 1 of panel B in Table 2 shows that the relation between realized value premium variance, $v_{H,t}^2$, and the one-quarter-ahead value premium, HML_{t+1}, is positive but statistically insignificant. However, the coefficient of $v_{H,t}^2$ becomes marginally significant after we control for realized covariance of the value premium with stock market returns, $v_{MH,t}$ (row 2). Because $v_{H,t}^2$ and $v_{MH,t}$ are negatively correlated with each other (as shown in Figure 1), these results suggest that the specification in row 1 of panel B also suffers from an omitted variable problem.

In row 3 of Table 2 we estimate the two equations jointly. We use a constant, $v_{M,t}^2$, and $v_{MH,t}$ as instrumental variables for the stock return equation and a constant, $v_{H,t}^2$, and $v_{MH,t}$ for the value premium equation. Thus the equation system is just-identified and the point estimates are identical to those reported in row 2. Note that, from row 3 on, we report the R-squared rather than the adjusted R-squared (as in rows 1 and 2) in the column under $\overline{R}^2$. In row 4 we impose the ICAPM restrictions that the constant terms are zero in both equations. The restrictions can be tested using Hansen’s (1982) J-test, which has a χ^2 distribution with 2 degrees of freedom. The J-test statistic is essentially zero, indicating that the restrictions cannot be rejected at any conventional significance level. Row 5 shows

[6] Because of the correlation between $v_{M,t}^2$ and $v_{MH,t}$, there is a potential concern over multicollinearity. However, multicollinearity cannot explain our results because it usually leads to lower *t*-statistics, in contrast with the increase of t-statistics when both variables are included. Moreover, the characteristic-root-ratio test proposed by Belsley, Kuh, and Welsch (1980) confirms that multicollinearity is unlikely to plague our results.

that we cannot reject the restrictions that the risk prices are equal across assets, and row 6 shows that we cannot reject the restrictions of no intercepts and the equal risk prices across assets. As expected, imposing the ICAPM restrictions improves the estimation efficiency and the standard errors in the restricted specifications are substantially smaller than those reported in row 3. After imposing all the ICAPM restrictions, row 6 shows that the slope coefficients are significant at the 1% level. Our results provide strong support for a positive risk-return tradeoff in the stock market after controlling for changes in investment opportunities, as proxied by the value premium.

Early authors, e.g., Fama and French (1989) and Campbell (1987), find that the dividend yield, the default premium, the term premium, and the stochastically detrended risk-free rate forecast stock market returns. Ferson and Harvey (1999) show that these variables also have some predictive power for the value premium. One possibility is that these variables comove with the variance and covariance terms in equation (3) at the business-cycle frequency. To address this issue, we include them as instrumental variables, in addition to those used in row 6 of Table 2. Row 7 shows that the model is not rejected at the 20% significance level, suggesting that the stock return predictability documented by early authors is indeed consistent with the ICAPM.

Lettau and Ludvigson (2001a) argue that the consumption-wealth ratio, CAY_t, is a strong predictor of stock market returns. If we also add CAY_t to the instrumental variable set (row 8, Table 2), only at the 5% significance level is the model not rejected; however, the other results are very similar to those reported in rows 6 and 7. Therefore, again, our results suggest that the value premium reflects intertemporal pricing, although it might be a noisier measure of investment opportunities than some other stock return predictors proposed in the literature.

Figure 1 shows that realized variances and covariance exhibit a big spike around the latest recession in our sample, during which stock prices first increased and then fell sharply. To investigate whether this seemingly unusual episode has any special effect on our inference, we analyze a shorter sample spanning the period 1963:Q3 to 1997:Q4 and report the results in rows 9 and 10 of Table 2, which have the same specifications as those in rows 7 and 8, respectively. We find that the results are very similar to those obtained using the full sample.

C. The Value Premium and Other Proxies of Investment Opportunities

Guo and Whitelaw (2006) use the consumption-wealth ratio, CAY_t, and the stochastically detrended risk-free rate, $RREL_t$, as proxies for investment opportunities. Guo and Savickas (2006) find that, when combined with stock market variance, IV_t—a measure of value-weighted idiosyncratic variance—forecasts stock market returns possibly because it is a proxy for realized variance of a risk factor omitted from the CAPM. We find that the forecasting power of the variables $v_{M,t}^2$ and $v_{MH,t}$ is qualitatively unchanged in the presence of $RREL_t$. Interestingly, the variable $v_{MH,t}$ loses its predictive power after we control for CAY_t or IV_t, while the effect of $v_{M,t}^2$ on expected stock returns remains positive and highly significant. Our results suggest that the value premium is related to the alternative measures of investment opportunities. For brevity, we do not report the details of the regression analysis here but they are available on request.

D. Conditional Value Premium

Consistent with equation (2), $v_{H,t}^2$ has some forecasting power for the value premium when combined with $v_{MH,t}$ (row 2 of Table 2). This result suggests that predictable variation in the value premium documented in some early studies—e.g., Ferson and Harvey (1999)—might be consistent with intertemporal pricing. To address this issue, in Table 3 we compare the forecasting power of $v_{H,t}^2$

with alternative measures of investment opportunities, namely, $RREL_t$, CAY_t, and IV_t.[7] The effect of $v_{H,t}^2$ remains positive and marginally significant after controlling for $RREL_t$ (row 1) and CAY_t (row 2). However, it becomes insignificant when combined with IV_t (row 3).

[Insert Table 3 Here]

In an earlier version of the paper, we showed that under some conditions the expected stock market return and expected value premium are linear functions of $v_{M,t}^2$ and $v_{H,t}^2$ and that such a specification holds even if the value premium is an imperfect measure of investment opportunities. Row 4 of Table 3 presents the regression results using $v_{M,t}^2$ instead of $v_{MH,t}$ in the forecasting equation. As expected, the alternative specification appears to provide a better fit for the value premium.[8] Now the effect of $v_{H,t}^2$ is positive and significant at the 5% level; and the effect of $v_{M,t}^2$ is negative and significant at the 5% level. Also, the adjusted R-squared is 4.9%, which is noticeably higher than the 3.9% reported in row 2 of Table 2. The coefficient of $v_{M,t}^2$ is negative because of its negative correlation with $v_{MH,t}$ (Table 1), which in turn is positively correlated with the value premium.

The forecasting power of $v_{H,t}^2$ (as in row 4 of Table 3) is very similar to that of IV_t, as reported by Guo and Savickas (2006). These authors show that IV_t and $v_{M,t}^2$ jointly have strong predictive power for the value premium; moreover, while $v_{M,t}^2$ is negatively correlated with the one-quarter-ahead

[7] The term premium, the default premium, and the dividend yield (as used by Ferson and Harvey (1999)) do not provide additional information about the future value premium, and including them does not change our results in any qualitative manner. For brevity, these results are not reported here but they are available on request.

[8] The two variables also have significant forecasting power for stock market returns in and out of sample. For brevity, these results are not reported here but they are available on request.

value premium, the relation is positive for IV_t. To formally address this issue, we also include IV_t in the forecasting equation, together with $v_{H,t}^2$ and $v_{M,t}^2$. Row 7 shows that, while the coefficient of $v_{M,t}^2$ remains significantly negative, the coefficients of both IV_t and $v_{H,t}^2$ become insignificant, indicating that the two variables indeed capture common variations in the value premium. This result should not be too surprising because Guo and Savickas (2006) point out that, by construction, IV_t is a proxy for realized variance of a risk factor omitted from the CAPM, which could be the value premium. However, by contrast with IV_t, controlling for $RREL_t$ (row 5) or CAY_t (row 6) does not affect our results in any qualitative manner.

[Insert Figure 2 Here]

Figure 2 plots the fitted expected value premium (as based on the estimation results of the benchmark ICAPM reported in row 6, Table 2). It tends to increase sharply just before or during the business recessions dated by NBER, as denoted by shaded areas. Interestingly, in a concurrent paper, Chen, Petkova, and Zhang (2007) also find that the conditional value premium tends to be countercyclical, although these authors use a very different estimation method. This new finding lends support to Zhang's (2005) full-fledged equilibrium model, in which value stocks are riskier than growth stocks during an economic downturn, when the price of risk is high. In Section V, we elaborate on the point that the conditioning information is important for understanding the value anomaly.

E. The Value Premium Constructed with Small and Big Stocks

If the value anomaly reflects intertemporal pricing, we expect to find very similar results using the value premium constructed with both small and big stocks. To investigate this issue, we obtain from Kenneth French the daily return data for six portfolios, which are the intersections of two independent sorts—size (small and big) and the book-to-market equity ratio (high, median, and low).

We find similar results using realized variance of the value premium constructed with small and big stocks. We do not report these results here but they are available on request.

III. Bivariate GARCH Model

A. Empirical Specifications

Several studies, e.g., Christensen and Prabhala (1998) and Fleming (1998), find that realized variance is not an efficient measure of conditional variance. To address this issue, in this section we estimate equations (1) and (2) using the more elaborate bivariate GARCH models, which might provide a better measure for the conditional second moments than the simple realized variance model.[9] Again, we rewrite equations (1) and (2) in the realized return form:

$$(4)\qquad \begin{aligned} R_{t+1} &= \alpha_R + \gamma_{MM}\sigma^2_{M,t} + \gamma_{HM}\sigma_{MH,t} + \varepsilon_{M,t+1} \\ HML_{t+1} &= \alpha_H + \gamma_{MH}\sigma_{MH,t} + \gamma_{HH}\sigma^2_{H,t} + \varepsilon_{H,t+1} \end{aligned},$$

where $\varepsilon_{M,t+1}$ and $\varepsilon_{H,t+1}$ are shocks to the market return and the value premium, respectively.

We use the asymmetric dynamic covariance (ADC) model proposed by Kroner and Ng (1998). These authors show that it is very flexible in describing the dynamics of covariance terms because it nests several commonly used multivariate GARCH models. In the ADC model, the dynamics of variances and covariance is governed by the following equation system:

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[9] In an earlier version of the paper, we formally investigate the relative performance of the realized variance model and the GARCH model using the Monte Carlo simulation. In particular, we first estimate the bivariate GARCH model using daily return data. We then use the estimated GARCH model to generate simulated daily data, which are used to estimate the ICAPM. We find that both the quarterly realized variance model and the monthly GARCH model provide reliable inference for the risk-return tradeoff, while the GARCH model performs somewhat better. For brevity, we do not report these results here but they are available on request.

$$
(5)\qquad
\begin{aligned}
&\sigma_{M,t}^{2} = \theta_{MM,t+1} \\
&\sigma_{H,t}^{2} = \theta_{HH,t+1} \\
&\sigma_{MH,t} = \rho_{MH}\sqrt{\theta_{MM,t+1}\theta_{HH,t+1}} + \phi_{MH}\theta_{MH,t+1} \\
&\theta_{ij,t+1} = \omega_{ij} + b_i' H_t b_j + a_i' \begin{bmatrix} \varepsilon_{M,t} \\ \varepsilon_{H,t} \end{bmatrix} \begin{bmatrix} \varepsilon_{M,t} & \varepsilon_{H,t} \end{bmatrix} a_j + g_i' \begin{bmatrix} \eta_{M,t} \\ \eta_{H,t} \end{bmatrix} \begin{bmatrix} \eta_{M,t} & \eta_{H,t} \end{bmatrix} g_j,\ i, j \in (H, M)
\end{aligned}
,
$$

where H_t is the conditional variance-covariance matrix:

$$
(6)\qquad H_t = \begin{bmatrix} h_{MM,t} & h_{MH,t} \\ h_{MH,t} & h_{HH,t} \end{bmatrix} = \begin{bmatrix} \sigma_{M,t-1}^{2} & \sigma_{MH,t-1} \\ \sigma_{MH,t-1} & \sigma_{H,t-1}^{2} \end{bmatrix}.
$$

Glosten, Jagannathan, and Runkle (1993), among many others, find that a negative return shock leads to a higher subsequent volatility than does a positive return shock of the same magnitude. This asymmetric effect can be captured by the term $\begin{bmatrix} \eta_{M,t} \\ \eta_{H,t} \end{bmatrix} = \begin{bmatrix} \max[0, -\varepsilon_{M,t}] \\ \max[0, -\varepsilon_{H,t}] \end{bmatrix}$ in equation (5). ρ_{MH} and ϕ_{MH} are scalar parameters and the other parameters can be written in matrix forms:

$$
(7)\qquad
\begin{aligned}
&W = C'C = \begin{bmatrix} \omega_{MM} & \omega_{MH} \\ \omega_{MH} & \omega_{HH} \end{bmatrix}, \quad A = \{a_M, a_H\} = \begin{bmatrix} a_{MM} & a_{MH} \\ a_{HM} & a_{HH} \end{bmatrix} \\
&B = \{b_M, b_H\} = \begin{bmatrix} b_{MM} & b_{MH} \\ b_{MH} & b_{HH} \end{bmatrix}, \quad G = \{g_M, g_H\} = \begin{bmatrix} g_{MM} & g_{MH} \\ g_{HM} & g_{HH} \end{bmatrix}
\end{aligned}
,
$$

where W is positive definite and C is a 2 × 2 symmetric matrix. Our notations in equation (7) reflect the fact that matrixes W and B are symmetric but matrixes A and G are not.

Kroner and Ng (1998) show that, if matrixes A and B are diagonal and ϕ_{MH} is equal to 0, the ADC model becomes the asymmetric version of the constant conditional correlation model, as used by Scruggs (1998), for example. Also, if ρ_{MH} is equal to 0 and ϕ_{MH} is equal to 1, then the ADC model reduces to the asymmetric version of the popular BEKK model proposed by Engle and Kroner (1995), which, as we show below, seems to apply in this study.

We estimate the GARCH model using the quasi-maximum likelihood (QML) method. Bollerslev and Woodridge (1992) show that QML parameter estimates can be consistent, even though the conditional log-likelihood function assumes normality while stock returns are known to be skewed and leptokurtic. Nevertheless, we find similar results using the maximum likelihood estimation (MLE) method by assuming a t distribution or a normal distribution. Given a sample of T observations of the return vector, the parameters of the bivariate GARCH model are estimated by maximizing the conditional log-likelihood function:

(8) $$L = \sum_{t=1}^{T} l_t(P) = \sum_{t=1}^{T} (-\log(2\pi) - 0.5\log|H_t| - 0.5\varepsilon_t^{'} H_t^{-1} \varepsilon_t),$$

where P denotes the vector of all the parameters to be estimated. Nonlinear optimization techniques are used to calculate the maximum likelihood estimates based on the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm.

The ADC model should be estimated under some parameter restrictions to ensure that the covariance matrix is positive definite. It is possible to impose the constraint $|\rho_{mv}| + |\phi_{mv}| < 1$ in the model. To serve a similar purpose, Scruggs and Glabadanidis (2003) propose penalizing the likelihood function whenever the covariance matrix is not positive definite, which we did in our study. While such treatment might lose the continuity of the likelihood function, it gains the ability to impose a less restrictive constraint and avoid the possibility of a non-positive definite covariance matrix. Also, imposing a penalty in the likelihood function might result in a function with multiple local optima. In this case, it is important to restart the optimization routine at several different starting points to ensure that the estimated parameters correspond to the global maximum of the likelihood function. All our results follow this procedure.

[Insert Table 4 Here]

We focus mainly on the period January 1963 to December 2005 because, as mentioned in footnote 2 and confirmed in this study, the value premium is a poor proxy for investment opportunities in the pre-1963 sample. Table 4 provides summary statistics of the excess stock market return and the value premium (in percentages) for the modern sample. Consistent with quarterly data in Table 1, the two variables are negatively correlated, with a correlation coefficient of –0.41. The Ljung-Box test indicates that the value premium is serially correlated.

B. Model Selection Tests

Kroner and Ng (1998), among others, argue that choosing a parsimonious GARCH specification is important for the asset pricing tests because they critically depend on the covariance matrix estimates. In fact, their ADC model was originally proposed to facilitate the model selection (Kroner and Ng (1998), p. 833). A parsimonious data-determined model is desirable also because the number of observations is limited, while a large amount of the data is required to yield precise estimates of GARCH models. Hence, it is important in this study to impose statistically acceptable constraints and reduce the redundant parameters.

[Insert Table 5 Here]

The model selection test follows the general-to-specific approach. As in Scruggs (1998) and Scruggs and Glabadanidis (2003), we first look at the second-moment modeling. The results, which are reported in Table 5, can be easily summarized as follows. Using the full-fledged bivariate ADC model as the alternative hypothesis, we overwhelmingly reject the null model of the pooling of two univariate GARCH specifications (panel A). By contrast, we fail to reject the more restrictive, and yet quite general, ABEKK model at the 10% level (panel B). Also, for the BEKK model, panel C shows that the null hypothesis of symmetry is strongly rejected. Because the ADC model involves more parameters and thus has poorer convergence properties, we hereafter focus on the ABEKK model in the remaining discussion, although we find similar results using the ADC model.

We then turn to the model selection test on the first-moment modeling for the ABEKK model. We first test the null hypothesis that the slope parameters are jointly zero in equation (4) or $\gamma_{MM} = \gamma_{MH} = \gamma_{HM} = \gamma_{HH} = 0$. Panel D of Table 5 shows that these restrictions are rejected at the 1% significance level, indicating that conditional variance and covariance terms are significant determinants of the excess stock market return and the value premium. However, consistent with the results obtained from the realized variance model, panels E and F show that we fail to reject the ICAPM restrictions at the conventional significance level.

C. Estimation Results

Table 6 presents the estimation results of the mean equations. We use the percentage return in the estimation; to make them comparable with the results in Table 2, we scale the constant terms by 1/100 and the slope parameters by 100. For comparison with early studies, we first report in panel A the estimation results of the pooling univariate asymmetric GARCH model—i.e., we restrict the interaction terms between the stock market return and the value premium to be zero in equations (4) and (5). For the excess stock market return equation, the conditional return is positively related to the conditional variance with a point estimate of 0.87; however, the relation is statistically insignificant at the 10% level. Similarly, we find a positive but insignificant risk-return relation for the value premium. Nevertheless, such a result should be interpreted with caution because the specification potentially suffers from an omitted variable problem, which we discuss next.

[Insert Table 6 Here]

Panel B of Table 6 presents the estimation results using the ABEKK model. In the unrestricted specification (row 2), only the slope parameters in the value premium equation are significant at the 10% level. Because the slope parameters are jointly significant (panel D of Table 5), this result suggests that our estimation is not efficient. One way to address this issue, as we have learned from the realized variance model reported in Table 2, is to impose the restrictions dictated by Merton's (1973) ICAPM. As expected, row 4 shows that the slope parameters in the mean equations are statistically significant at the 1% level after we impose the ICAPM restrictions of zero constant terms and the same risk prices across assets.

In the benchmark model (row 4, Table 6), the price of stock market risk, γ_M, has a point estimate of 4.74 and a standard error of 1.21. It appears to be quite reasonable because Mehra and Prescott (1985), for example, suggest a plausible range 1 to 10. Interestingly, it is also strikingly similar to the point estimate of 4.93 reported by Guo and Whitelaw (2006), who use CAY_t as a proxy for investment opportunities. This is mainly because, as explained in footnote 3, $v_{MH,t}$ and CAY_t are likely to capture the common variations of stock market returns.

[Insert Figure 3 Here]

Figure 3 plots the fitted values of conditional stock market variance (dashed line), conditional value premium variance (solid line), and conditional covariance between the stock market return and the value premium (thick solid line) from the benchmark estimation. The pattern is very similar to that presented in Figure 1. For example, conditional stock market variance is negatively correlated with the conditional covariance between the market return and the value premium, indicating that the omitted variable problem is responsible for the negative risk-return tradeoff documented in the early studies. Figure 4 plots the fitted value premium for the benchmark specification. Again, the pattern is very similar to that obtained from the realized variance model, as presented in Figure 2. In particular, the conditional value premium tends to increase sharply just before or during the business recessions dated by NBER.

[Insert Figure 4 Here]

We also document substantial variation in the coefficient of conditional correlation between the stock market return and the value premium. This result confirms the finding of Scruggs and Glabadanidis (2003) that it is important to allow for a time-varying correlation coefficient in the ICAPM estimation. Moreover, in the benchmark model, most parameters in the matrices W, A, B, and G are statistically significant, indicating that it is important to allow for a time-varying variance-covariance matrix. For brevity, we do not report the details of these estimation results here but they are available on request.

D. Robustness Checks and Diagnostics Tests

Panel C of Table 4 reports the mean of fitted values of conditional variances and covariance based on the estimation results of the benchmark specification. They are very similar to the unconditional variance-covariance matrix of the excess stock market return and the value premium, as reported in panel B of Table 4.

Row 5 of Table 6 reports the estimation results of the ABEKK model for the early period July 1926 to December 1962. The risk price associated with the value premium has a negligible point estimate of -0.002, which is statistically insignificant at any conventional level. The price of stock market risk is again statistically significant; nevertheless, its point estimate of 2.20 is substantially smaller than the point estimate of 4.74 obtained from the modern period, as reported in row 4 of Table 6. These results confirm Campbell and Vuolteenaho's (2004) finding that in the early period the value premium is a poor proxy for investment opportunities. Row 6 shows that in the full sample spanning the period July 1927 to December 2005, the value premium risk is not priced but the price of the market risk is significantly positive. Because of the likely structural break in the value premium, we should interpret this result with caution.

Although we concentrate on a restricted ABEKK specification in the previous discussion, it is worth noting that we find similar results using the ADC model, as shown in panel C of Table 6. In the unrestricted model (row 7), we find that the risk prices are all positive, although most of them are statistically insignificant. By contrast, row 8 shows that the risk prices again become significant at the 1% level after imposing the ICAPM restrictions, which cannot be rejected at the conventional significance level. Moreover, the point estimates are very similar to those obtained using the benchmark ABEKK model.

We also estimate the restricted ABEKK model using the MLE method by assuming a t distribution and a normal distribution for the modern sample. We report the main results in rows 9 and 10, respectively, of Table 6. For the t distribution, the degree of freedom of the distribution has a point estimate of 9.14 and a standard error of 1.91. This result is consistent with the general belief that the distribution of stock returns is characterized by fat tails. Nevertheless, the other results are essentially the same as the benchmark ABEKK model. We reach the same conclusion for the normal distribution as well.

We repeat the above analysis using daily and weekly data. Again, our main finding that the loadings on the stock market return and the value premium carry a positive and significant risk premium holds well in the modern period. For brevity, these results are not reported here but are available on request.

Lastly, to evaluate the adequacy of the benchmark model, we conduct several specification tests on the standardized residuals ($\hat{\varepsilon}_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}, \quad i = M, H$) and standardized products of residuals ($\hat{\varepsilon}_{i,t} \hat{\varepsilon}_{j,t} = \varepsilon_{i,t}\varepsilon_{j,t} / h_{ij,t}, \quad i = M, H$). Specifically, we examine some moment conditions required for the consistency of QML estimates. The two mean standardized residuals are not significantly different from zero. However, the evidence is somewhat mixed for testing the null hypothesis that the mean of the products of the residuals is 1. The null cannot be rejected for $\hat{\varepsilon}_{M,t} \hat{\varepsilon}_{M,t}$ and $\hat{\varepsilon}_{H,t} \hat{\varepsilon}_{H,t}$ but can be rejected for the cross-product, $\hat{\varepsilon}_{M,t} \hat{\varepsilon}_{H,t}$. We also note that the skewness and kurtosis for the standardized residuals is much lower than the skweness and kurtosis for the value premium but not for the stock market return. The Ljung-Box test indicates that the autocorrelation is still present in the residuals of the value premium equation. (Recall that the original value premium series contains autocorrelation.) Overall, these results suggest that, while the model provides a reasonable description of the data, there is still room for improvement.

IV. International Evidence

To address the question of data mining, Fama and French (1998) investigate the value premium for major international equity markets constructed from MSCI (the Morgan Stanley Capital International) data. They have two main findings. First, the value premium is pervasive in major international equity markets. Second, the value premium appears to be a priced risk factor omitted from the CAPM. In this section we estimate the bivariate GARCH model using the Fama and French international data for the period January 1975 to December 2005.

Without the loss of generality, we focus on the world market as well as the other G7 countries, namely, Canada, France, Germany, Italy, Japan, and the U.K. The world market portfolios are especially relevant because they are the most diversified: For example, Fama and French (1998) use the world market return and value premium as risk factors in their international ICAPM. We also expect to uncover similar patterns for each of the other G7 countries because Fama and French (1998) find that the country-specific stock market return and value premium move closely with their world market counterparts.

[Insert Table 7 Here]

For brevity, we consider only the ABEKK model because, consistent with U.S. evidence, it also provides a good description for all the international markets that we considered. In the estimation we also impose the ICAPM restrictions: $\gamma_{MM} = \gamma_{MH}$, $\gamma_{HM} = \gamma_{HH}$, and $\alpha_R = \alpha_H = 0$, which we fail to reject using the log likelihood ratio test. Table 7 shows that international evidence is quite consistent with that documented in U.S. data. For the world market, the price of market risk, γ_M, is significantly positive, with a point estimate of 3.16. Similarly, the risk price for the value premium, γ_H, is significantly positive, with a point estimate of 8.18. We also find similar results for the individual markets. Except for Italy, the parameter γ_H is positive and statistically significant at least at the 10% level for all the other G7 countries. Similarly, the parameter γ_M is always positive, and it is significant at least at the 10% level for France, Germany, Japan, and the U.K. Thus, the international evidence provides further support for the conjecture that the value premium is a proxy for investment opportunities.

V. Some Discussions

In the post-1963 sample, the CAPM fails to explain the value premium. Lakonishok, Shleifer, and Vishny (1994) argue that the value premium reflects mispricing: Investors tend to overestimate future earnings of growth stocks but underestimate future earnings of value stocks. MacKinlay (1995) attributes the value premium to data mining. By contrast, Fama and French (1996, 1998) suggest that the value premium should reflect systematic risk because it is a pervasive phenomenon in both the U.S. and international stock markets. Following Fama and French's conjecture, some authors have proposed several risk-based explanations for the value premium.

First, because high book-to-market equity ratios are typical of stocks that are relatively distressed, Fama and French (1996) suggest that the value premium might reflect a distress risk. Consistent with this hypothesis, Fama and French (1995) find that the effect of the distress risk is more pronounced during business recessions than during business expansions.

Second, Campbell and Vuolteenaho (2004) estimate a variant of Campbell's (1993) ICAPM. They find that in the post-1963 sample, the value premium has a negative market beta because of its large *positive* loadings on the discount-rate shock. Moreover, the expected value premium is positive because of its positive loadings on the cash-flow shock, which carries a much higher risk price than does the discount-rate shock.[10] However, Chen and Zhao (2005) and Liu and Zhang (2007) argue that their results might be sensitive to the choice of state variables.

Lastly, Jagannathan and Wang (1996), Lettau and Ludvigson (2001b), and Petkova and Zhang (2005) have empirically investigated conditional asset pricing models by using conditioning variables to scale the betas. They find that value stocks are riskier—e.g., have a higher market beta or consumption beta—than growth stocks during economic recessions when the price of risk is high. Therefore, the value premium could have a positive mean, even though its unconditional market beta is negative, as reported in Table 1. Zhang (2005) also develops a full-fledged equilibrium model and shows that under reasonable parameterizations, the conditioning information is crucial for understanding the value anomaly. However, Lewellen and Nagel (2006) argue that the commonly used conditional models can account for only a small fraction of the observed value premium.

This paper documents a new finding of a significantly positive relation between the value premium and its conditional variance. Because the conditional variance of the value premium is strongly countercyclical (as shown in Figures 1 and 3), the expected value premium tends to be high during business recessions and to be low during business expansions (as shown in Figures 2 and 4). These results lend support to the conditional asset pricing models advocated by Jagannathan and Wang (1996), Lettau and Ludvigson (2001b), Petkova and Zhang (2005), and Zhang (2005), which predict a countercyclical conditional value premium. Our results are quantitatively different from those in Lewellen and Nagel (2006) presumably because we use more sophisticated empirical methods.

VI. Conclusion

This paper estimates a variant of Merton's (1973) ICAPM using the value premium as a proxy for time-varying investment opportunities. In contrast with many early authors, we uncover a positive and significant risk-return tradeoff after controlling for covariance of the stock market return with the value premium. We also document a new finding on a significantly positive relation between the value premium and its conditional variance. These results suggest that we cannot fully attribute the value premium to irrational pricing or data mining.

Our results also shed light on time-series stock market return predictability. We find that it cannot by fully attributed to irrational pricing or data mining for three reasons. First, existing economic

[10] Lettau and Wachter (2007) also propose an equilibrium model to explain the stylized fact that the value premium has a positive mean but a negative market beta. The main economic intuition of their model is similar to that of Campbell and Vuolteenaho (2004).

theories have provided guidance for identifying predictive variables, i.e., conditional variances and covariances of the risk factors in Merton’s (1973) ICAPM. Second, despite its simplicity, our analysis shows that the theoretically motivated variables forecast stock market returns in and out of sample. Third, many financial variables forecast stock returns mainly because of their close correlation with conditional variances and covariances of stock market returns and other risk factors.

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TABLE 1

Summary Statistics of Quarterly Data

Table 1 reports summary statistics for the excess stock market return, R_t; the value premium, HML_t; realized stock market variance, $v^2_{M,t}$; realized variance of the value premium, $v^2_{H,t}$; and realized covariance between the stock market return and the value premium, $v_{MH,t}$. The sample spans the period 1963:Q3 to 2005:Q4.

	R_t	HML_t	$v^2_{M,t}$	$v^2_{H,t}$	$v_{MH,t}$
Panel A Correlation Matrix					
R_t	1.000				
HML_t	-0.461	1.000			
$v^2_{M,t}$	-0.370	0.081	1.000		
$v^2_{H,t}$	-0.273	0.291	0.675	1.000	
$v_{MH,t}$	0.358	-0.190	-0.818	-0.931	1.000
Panel B Univariate Statistics					
Mean	0.015	0.014	0.005	0.001	-0.001
Standard Deviation	0.085	0.059	0.005	0.002	0.002
Autocorrelation	0.022	0.135	0.531	0.721	0.565

TABLE 2

Merton's (1973) ICAPM: Realized Variance Model

Table 2 reports the estimation results of Merton's (1973) ICAPM using the GMM:

(3)
$$R_{t+1} = \alpha_M + \gamma_{MM} v^2_{M,t} + \gamma_{HM} v_{MH,t} + \varepsilon_{M,t+1}$$
$$HML_{t+1} = \alpha_H + \gamma_{MH} v_{MH,t} + \gamma_{HH} v^2_{H,t} + \varepsilon_{H,t+1},$$

where R_{t+1} is the excess stock market return; HML_{t+1} is the value premium; $v^2_{M,t}$ is realized stock market variance; $v_{MH,t}$ is realized covariance between the stock market return and the value premium; $v^2_{H,t}$ is realized variance of the value premium; and $\varepsilon_{M,t+1}$ and $\varepsilon_{H,t+1}$ are shocks to the stock market return and the value premium, respectively. Unless otherwise indicated, we use the quarterly sample spanning the period 1963:Q3 to 2005:Q4. The heteroskadesticity-corrected standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. In the column under $\overline{R}^2$, the adjusted R-squared is reported in rows 1 and 2 and the R-squared is reported in the other rows. The two equations are estimated separately in rows 1 and 2 and jointly in the other rows. The system is just identified in row 3: We use a constant, $v^2_{M,t}$, and $v_{MH,t}$ as instrumental variables for the stock market return equation and use a constant, $v^2_{H,t}$, and $v_{MH,t}$ for the value premium equation. We impose the restriction of zero intercept in row 4, the restriction of the same risk prices in row 5, and both restrictions in rows 6 to 8. We use the same instrumental variables in rows 4 to 6 as in row 3. We also include the default premium, the term premium, the stochastically detrended risk-free rate, and the dividend yield as instrumental variables in row 7. Row 8 also includes the consumption-wealth ratio by Lettau and Ludvigson (2001a) as an instrumental variable. We report Hansen's (1982) J-test in the column under J-Test, with the p-value in parentheses. Rows 9 and 10 have the same specifications as rows 7 and 8, respectively, but are estimated for the sample period 1963:Q3 to 1997:Q4.

	Panel A Stock Market Returns				Panel B the Value Premium				J-Test
	α_R	γ_{MM}	γ_{HM}	$\overline{R}^2$	α_H	γ_{MH}	γ_{HH}	$\overline{R}^2$	
1	0.002	2.713*		0.016	0.007		4.860	0.017	
	(0.008)	(1.475)			(0.005)		(3.459)		
2	-0.004	7.725***	12.386**	0.048	0.007	11.508	18.246*	0.039	
	(0.008)	(2.559)	(5.461)		(0.005)	(7.428)	(9.988)		
3	-0.004	7.725***	12.386**	0.059	0.007	11.508	18.246*	0.051	
	(0.008)	(2.559)	(5.461)		(0.005)	(7.428)	(9.988)		
4		7.725***	12.386**	0.059		11.508	18.246**	0.051	$\chi^2(2)$ =0.00
		(2.018)	(4.948)			(7.155)	(8.908)		(1.00)
5	-0.004	8.160***	13.544***	0.059	0.008	8.160***	13.544***	0.050	$\chi^2(2)$ =0.37
	(0.008)	(2.500)	(5.050)		(0.005)	(2.500)	(5.050)		(0.83)
6		8.162***	13.547***	0.059		8.162***	13.547***	0.050	$\chi^2(4)$ =0.37
		(1.750)	(3.806)			(1.750)	(3.806)		(0.99)
7		7.748***	12.792***	0.059		7.748***	12.792***	0.050	$\chi^2(12)$ =15.74
		(1.693)	(3.723)			(1.693)	(3.723)		(0.20)
8		7.859***	13.358***	0.059		7.859***	13.358***	0.050	$\chi^2(14)$ =23.39
		(1.696)	(3.712)			(1.696)	(3.712)		(0.05)
9		10.303***	13.852***	0.043		10.303***	13.852***	0.006	$\chi^2(12)$ =17.09
		(2.074)	(4.976)			(2.074)	(4.976)		(0.15)
10		10.988***	13.748***	0.045		10.988***	13.748***	0.005	$\chi^2(14)$ =23.56
		(2.093)	(4.908)			(2.093)	(4.908)		(0.05)

TABLE 3

Forecasting Quarterly Value Premium

Table 3 reports the OLS regression results of forecasting the one-quarter-ahead value premium using some predetermined variables over the period 1963:Q3 to 2005:Q4. The heteroskadesticity-corrected standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. $v_{M,t}^2$ is realized stock market variance; $v_{H,t}^2$ is realized variance of the value premium; $v_{MH,t}$ is realized covariance between the stock market return and the value premium; $RREL_t$ is the stochastically detrended risk-free rate; CAY_t is the consumption-wealth ratio proposed by Lettau and Ludvigson (2001a); and IV_t is a measure of idiosyncratic variance used in Guo and Savickas (2006).

	$v_{M,t}^2$	$v_{MH,t}$	$v_{H,t}^2$	$RREL_t$	CAY_t	IV_t	$\bar{R}^2$
1		11.758	18.683*	2.847**			0.056
		(7.367)	(9.827)	(1.392)			
2		11.008	16.843*		-0.352		0.040
		(7.464)	(9.951)		(0.315)		
3		11.619	16.985			0.231	0.034
		(7.444)	(10.548)			(0.919)	
4	-3.275**		10.455**				0.049
	(1.429)		(4.726)				
5	-3.045**		10.186**	2.407*			0.059
	(1.422)		(4.615)	(1.393)			
6	-3.166**		9.476**		-0.341		0.049
	(1.431)		(4.623)		(0.315)		
7	-3.945**		5.391			1.031	0.052
	(1.707)		(6.179)			(1.079)	

TABLE 4
Summary Statistics of Monthly Data

Table 4 reports summary statistics of the excess stock market return, R_t, and the value premium, HML_t, in percentages. Panel B reports the unconditional variance-covariance matrix in the upper triangle and the correlation coefficient in the lower triangle. Panel C reports the conditional variances and covariance, which are based on estimation of the benchmark ABEKK model reported in row 4, Table 6. $\sigma^2_{M,t}$ is stock market variance, $\sigma^2_{H,t}$ is variance of the value premium, and $\sigma_{MH,t}$ is covariance of the stock market return with the value premium. The sample spans the period January 1963 to December 2005. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Panel A Summary Statistics

	Mean	Standard Deviation	Skewness	Kurtosis	Ljung-Box statistics		
					Q1	Q6	Q12
R_t	0.481	4.409	–0.505	5.065	1.427	5.587	8.878
HML_t	0.457	2.911	0.005	5.505	8.930***	14.405**	17.422

Panel B Unconditional Covariance Matrix

	R_t	HML_t
R_t	19.435	–5.232
HML_t	–0.408	8.472

Panel C Mean of Conditional Variances and Covariance

$\sigma^2_{M,t}$	$\sigma_{MH,t}$	$\sigma^2_{H,t}$
19.103	–5.177	8.259

TABLE 5

Specification Tests for GARCH Models

Table 5 reports the specification tests of the GARCH model described in equations (4) through (7). The sample spans the period January 1963 to December 2005.

Null hypothesis	DF	LR	P-Value
Panel A Pooling Univariate GARCH Model vs. ADC Model			
H_0: No Interaction Term	10	129.40	0.00
Panel B ABEKK model vs. ADC Model			
H_0: $\rho_{MH} = 0$ and $\phi_{MH} = 1$	2	4.30	0.12
Panel C BEKK Model vs. ABEKK Model			
H_0: $g_{MM} = g_{MH} = g_{HM} = g_{HH} = 0$	4	28.28	0.00
Panel D Constant Equity Premium and Value Premium in ABEKK Model			
H_0: $\gamma_{MM} = \gamma_{MH} = \gamma_{HM} = \gamma_{HH} = 0$	4	18.88	0.00
Panel E No Constant Terms in ABEKK Model			
H_0: $\alpha_{ER} = \alpha_{HML} = 0$	2	0.12	0.94
Panel F Equal Risk Prices Across Assets in ABEKK Model			
H_0: $\gamma_{MM} = \gamma_{MH}$, $\gamma_{HM} = \gamma_{HH}$ $\alpha_{ER} = \alpha_{HML} = 0$	4	4.79	0.31

TABLE 6

Merton's (1973) ICAPM: Bivariate GARCH Model

Table 6 reports the estimation results of Merton's (1973) ICAPM using various bivariate GARCH models described in equations (4) through (7). We report standard errors in parentheses. Unless otherwise indicated, we use the QML method and the monthly sample spanning the period January 1963 to December 2005. We use the sample period July 1926 to December 1962 in row 5 and the sample period July 1926 to December 2005 in row 6. The specifications in rows 9 and 10 are the same as those in row 4 except that we assume a t distribution in row 9 and a normal distribution in row 10. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. We report the log likelihood in the column under LL.

	Stock Market Returns			Value Premium			LL
	α_R	γ_{MM}	γ_{HM}	α_H	γ_{MH}	γ_{HH}	
				Panel A Pooling Univariate GARCH			
1	0.004	0.87		-0.001		6.71	-2681.59
	(0.004)	(2.29)		(0.003)		(4.12)	
				Panel B ABEKK Model			
2	0.000	6.00	10.91	0.001	19.04*	16.56*	-2619.05
	(0.623)	(4.35)	(6.57)	(0.229)	(10.90)	(9.50)	
3		5.40***	11.84*		16.49**	16.01***	-2619.11
		(1.87)	(6.33)		(7.72)	(5.04)	
4		4.74***	7.46***		4.74***	7.46***	-2621.44
		(1.21)	(1.95)		(1.21)	(1.95)	
5		2.20***	-0.002		2.20***	-0.002	-2385.97
		(0.84)	(0.016)		(0.84)	(0.016)	
6		2.52***	1.63*		2.52***	1.63*	-5075.29
		(0.584)	(0.897)		(0.584)	(0.897)	
				Panel C ADC Model			
7	-0.001	5.82	9.95	-0.002	17.22	19.20***	-2616.90
	(0.001)	(4.29)	(5.67)	(0.002)	(15.31)	(4.79)	
8		4.73***	8.22***		4.73***	8.22***	-2618.34
		(1.22)	(2.01)		(1.22)	(2.01)	
				Panel D ABEKK Model Using MLE Method			
9		4.97***	7.54***		4.97***	7.54***	–2603.72
		(1.16)	(1.83)		(1.16)	(1.83)	
10		4.74***	7.46***		4.74***	7.46***	-2621.44
		(1.15)	(1.71)		(1.15)	(1.71)	

TABLE 7

International Evidence

Table 7 reports the estimation results of the ABEKK specification of equations (4) through (7) by imposing the restrictions ρ_{MH} = 0 and ϕ_{MH} =1. We also impose the ICAPM restrictions $\gamma_{MM} = \gamma_{MH}$, $\gamma_{HM} = \gamma_{HH}$, and $\alpha_R = \alpha_H = 0$. The sample spans the period January 1975 to December 2005. The returns are denoted in local currencies for the G7 countries and in the U.S. dollar for the world market. ***, **, and * denote significance at the 1%, 5%, and 10% significance levels, respectively.

Country	γ_M	Standard Errors	γ_H	Standard Errors
Canada	1.509	1.354	2.009*	1.090
France	1.842*	0.964	2.112*	1.276
Germany	1.878**	0.823	2.702**	1.218
Italy	0.645	0.807	-0.704	1.065
Japan	1.897*	1.054	3.873***	1.207
UK	2.450**	1.241	2.658**	1.321
World	3.157***	1.184	8.179***	2.334

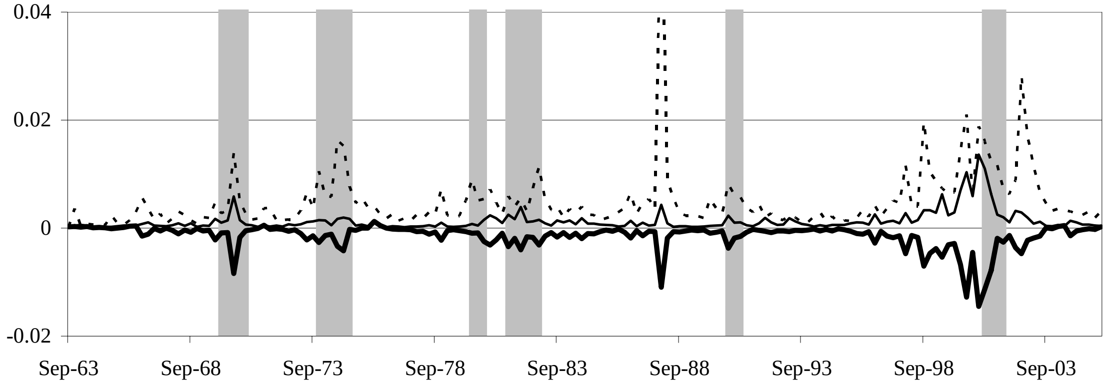

FIGURE 1
Quarterly Realized Variances and Covariance

Dashed line is realized stock market variance, $v_{M,t}^2$; solid line is realized value premium variance, $v_{H,t}^2$; and thick solid line is realized covariance of the market return with the value premium, $v_{MH,t}$. The quarterly sample spans the period 1963:Q3 to 2005:Q4. $v_{M,t}^2$ is the sum of squared daily excess stock market returns in quarter t; $v_{H,t}^2$ is the sum of squared daily value premium; and $v_{MH,t}$ is the sum of the cross-product of the daily excess stock market returns with the value premium. The shaded areas indicate business recessions dated by NBER.

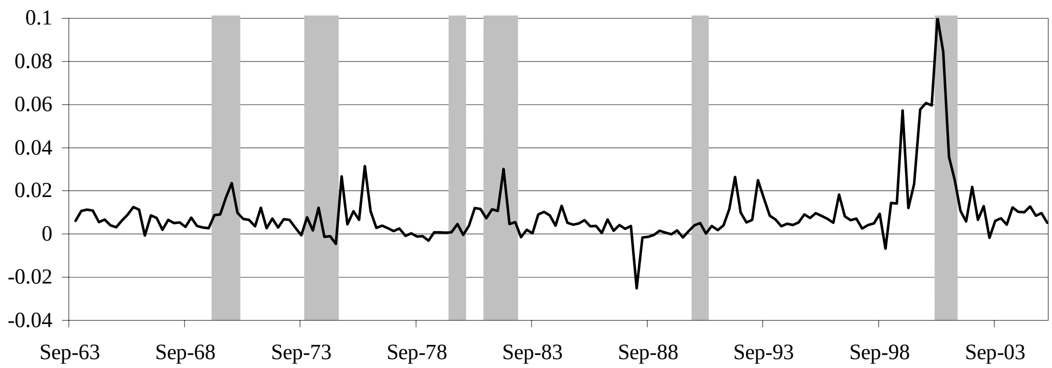

FIGURE 2
Quarterly Fitted Value Premium

Using quarterly data over the period 1963:Q3 to 2005:Q4, we estimate Merton's (1973) ICAPM

$$
(3) \quad \begin{aligned} R_{t+1} &= \alpha_M + \gamma_{MM} v_{M,t}^2 + \gamma_{HM} v_{MH,t} + \varepsilon_{M,t+1} \\ HML_{t+1} &= \alpha_H + \gamma_{MH} v_{MH,t} + \gamma_{HH} v_{H,t}^2 + \varepsilon_{H,t+1} \end{aligned},
$$

where R_{t+1} is the excess stock market return; HML_{t+1} is the value premium; $v_{M,t}^2$ is realized stock market variance; $v_{MH,t}$ is realized covariance between the market return and the value premium; $v_{H,t}^2$ is realized variance of the value premium; and $\varepsilon_{M,t+1}$ and $\varepsilon_{H,t+1}$ are shocks to the market return and the value premium, respectively. In the estimation we have imposed the ICAPM restrictions: $\alpha_M = \alpha_H = 0$, $\gamma_{MM} = \gamma_{MH} = \gamma_M$, and $\gamma_{HM} = \gamma_{HH} = \gamma_H$. The fitted value premium for quarter t+1 is equal to $\hat{\gamma}_{MH} v_{MH,t} + \hat{\gamma}_{HH} v_{H,t}^2$, where $\hat{\gamma}_{MH}$ and $\hat{\gamma}_{HH}$ are the estimated slope parameters. The shaded areas indicate business recessions dated by NBER.

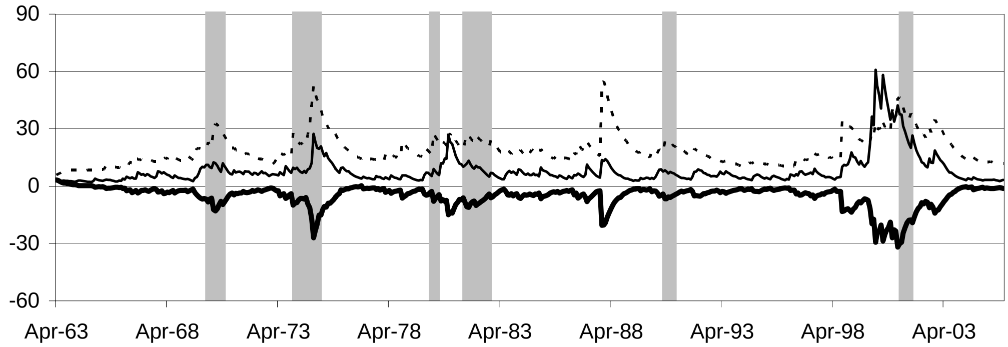

FIGURE 3

Monthly Conditional Variances and Covariance Estimated Using the Benchmark GARCH Model

Dashed line is conditional stock market variance, $\sigma^2_{M,t}$; solid line is conditional value premium variance, $\sigma^2_{H,t}$; and thick solid line is conditional covariance of the market return with the value premium, $\sigma_{MH,t}$. The monthly sample spans the period April 1963 to December 2005. We estimate the conditional second moments using the benchmark ABEKK model, in which we impose all the ICAPM restrictions. The shaded areas indicate business recessions dated by NBER.

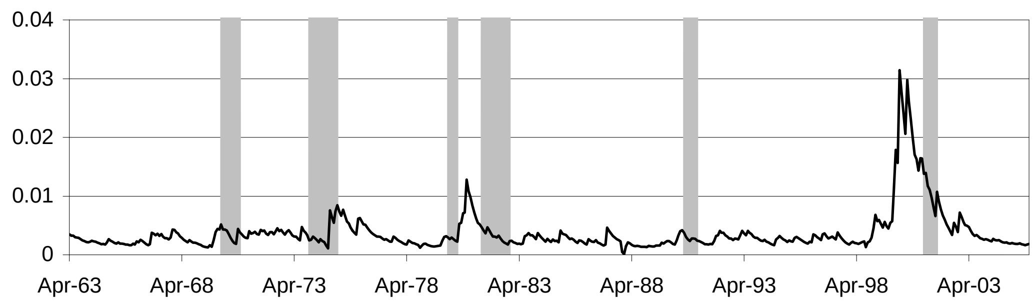

FIGURE 4

Monthly Fitted Value Premium

Using the monthly data over the period April 1963 to December 2005, we estimate Merton's (1973) ICAPM

(3)
$$\begin{aligned} R_{t+1} &= \alpha_R + \gamma_{MM}\sigma^2_{M,t} + \gamma_{HM}\sigma_{MH,t} + \varepsilon_{M,t+1} \\ HML_{t+1} &= \alpha_H + \gamma_{MH}\sigma_{MH,t} + \gamma_{HH}\sigma^2_{H,t} + \varepsilon_{H,t+1} \end{aligned},$$

where R_{t+1} is the excess stock market return; HML_{t+1} is the value premium; $\sigma^2_{M,t}$ is conditional stock market variance; $\sigma_{MH,t}$ is conditional covariance between the market return and the value premium; $\sigma^2_{H,t}$ is conditional variance of the value premium; and $\varepsilon_{M,t+1}$ and $\varepsilon_{H,t+1}$ are shocks to the market return and the value premium, respectively. We estimate the conditional second moments using the benchmark ABEKK model with the ICAPM restrictions: $\alpha_M = \alpha_H = 0$, $\gamma_{MM} = \gamma_{MH} = \gamma_M$, and $\gamma_{HM} = \gamma_{HH} = \gamma_H$. The fitted value premium for month $t+1$ is equal to $\hat{\gamma}_{MH}\sigma_{MH,t} + \hat{\gamma}_{HH}\sigma^2_{H,t}$, where $\hat{\gamma}_{MH}$ and $\hat{\gamma}_{HH}$ are the estimated slope parameters. The shaded areas indicate business recessions dated by NBER.