Investigating the Intertemporal Risk-Return Relation in International Stock Markets with the Component GARCH Model

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Abstract

Daily data and component GARCH (CGARCH) models strongly support a positive risk-return relation, in contrast to previous international results. Long-run volatility appears to be important in determining the conditional equity premium, but the evidence might be spurious.

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1. Introduction

Merton's (1973) intertemporal capital asset pricing model (ICAPM) stipulates a positive relation between conditional excess stock market returns and variance. However, many authors, (e.g., Glosten et al., 1993), document a negative relation in U.S. data.

This paper investigates the risk-return relation with Morgan Stanley Capital International (MSCI) data for 19 major international stock markets, including the world market. Our approach differs from previous studies of international data along three dimensions. First, our 30 years of daily data more precisely measures volatility and identifies the risk-return relation than does the shorter span of weekly data used by Theodossiou and Lee (1995) and Li. et al. (2005). Second, we use Engle and Lee's (1999) component GARCH (CGARCH) model that many authors have touted as a superior volatility model (Christoffersen et al. (2006)). Third, we distinguish the effects of the long- and short-run volatility components on returns, as in Engle and Lee (1999) and Adrian and Rosenberg (2006).

In contrast with previous evidence, we document a positive risk-return relation in most international stock markets. Statistical tests strongly reject standard GARCH models in favor of more elaborate CGARCH models, which provide slightly more evidence for a positive risk-return relation. Finally, long-run volatility appears to significantly determine the conditional equity premium while short-run volatility does not. We conjecture, however, that this last relation might be spurious.

2. Data

We use (approximately) 7600 daily and 1547 weekly MSCI gross excess total stock market returns for 19 international markets, including the world, from January 7, 1974 to August 29, 2003. Monthly dividends and interest rates are interpolated to daily and weekly frequencies to compute total excess returns. Summary statistics are omitted for brevity but are typical of excess equity returns.

3. Empirical Specifications

This paper uses Engle and Lee's (1999) asymmetric CGARCH model and the asymmetric GARCH of Glosten et al. (1993). The GARCH model is as follows:

$$\begin{aligned} r_{t+1} &= c + \lambda h_{t+1} + \varepsilon_{t+1} \\ \varepsilon_{t+1} &= h_{t+1} z_{t+1} \\ h_{t+1} &= \omega + \alpha\left(\varepsilon_t^2 - \omega\right) + \delta\left(D_t \varepsilon_t^2 - .5\omega\right) + \beta\left(h_t - \omega\right) \end{aligned} , \quad (1)$$

$$D_t = \left\{ \begin{matrix} 1 & if \;\; \varepsilon_t < 0 \\ 0 & otherwise \end{matrix} \right\}, \; \hat{\omega} = E\left[h_t\right], \; \hat{\alpha} > 0 \; , \; \hat{\beta} > 0 \; ,$$

where r_{t+1} is the excess return, h_{t+1} is conditional variance and z_t has a t distribution. δ captures the tendency for volatility to react more strongly to negative shocks. Merton's ICAPM requires that c equals zero — expected excess returns are proportional to conditional variance, where the factor of proportion is λ, the coefficient of relative risk aversion.

Engle and Lee's (1999) CGARCH model permits both a slowly mean reverting long-run component of conditional variance, q_t, and a more volatile, short-run component, $h_t - q_t$.

$$\begin{aligned} r_{t+1} &= c + \lambda h_{t+1} + \varepsilon_{t+1} \\ \varepsilon_{t+1} &= h_{t+1} z_{t+1} \\ h_{t+1} &= q_{t+1} + \alpha\left(\varepsilon_t^2 - q_t\right) + \delta_2\left(D_t \varepsilon_t^2 - .5 q_t\right) + \beta\left(h_t - q_t\right), \\ q_{t+1} &= \omega + \rho q_t + \phi\left(\varepsilon_t^2 - h_t\right) + \delta_1\left(D_t \varepsilon_t^2 - .5 h_t\right) \end{aligned} \quad (2)$$

where $\hat{\omega} = E\left[h_t\right]\left(1 - \hat{\rho}\right)$, $0 < \hat{\rho} < 1$, $\hat{\alpha} > 0$ and $\hat{\beta} > 0$. Christoffersen et al. (2006) find that distinguishing short- and long-run components enables CGARCH to describe volatility dynamics better than GARCH.

Equation (2) assumes the prices of risk for long- and short-run volatility are equal. Engle and Lee (1999), however, find that the long-run component is a more important determinant of the conditional equity premium than the short-run component. Adrian and Rosenberg (2006) develop an ICAPM in which both volatility components are priced factors. Therefore we consider a specification (CGARCH2L) in which the long- and short-run volatility components have different effects on returns (λ_2 and λ_1):

$$
\begin{aligned}
r_{t+1} &= c + \lambda_1 (h_{t+1} - q_{t+1}) + \lambda_2 q_{t+1} + \varepsilon_{t+1} \\
\varepsilon_{t+1} &= h_{t+1} z_{t+1} \\
h_{t+1} &= q_{t+1} + \alpha\left(\varepsilon_t^2 - q_t\right) + \delta_2\left(D_t \varepsilon_t^2 - .5 q_t\right) + \beta\left(h_t - q_t\right) \\
q_{t+1} &= \omega + \rho q_t + \phi\left(\varepsilon_t^2 - h_t\right) + \delta_1\left(D_t \varepsilon_t^2 - .5 h_t\right)
\end{aligned}
\quad (3)
$$

4. Empirical Results

4.1 Model Selection

We begin by selecting among our 3 candidate models: CGARCH2L, CGARCH and GARCH, with and without a constant term in the return equation. Likelihood ratio (LR) tests show that the constant is significant in about half the markets with the GARCH, CGARCH and CGARCH2L models, but simulations indicate that the test is significantly oversized. It is not clear if the constant should be restricted; we report results with and without the constant. The data clearly reject the parsimonious GARCH model in favor of the CGARCH model with either 1 or 2 lambdas, for all markets. The data also reject 1 lambda in favor of 2 lambdas for about half of the markets. Again, simulated data—calibrated to match the U.S. daily data with 2 lambdas—shows that the test has poor power against the null: the extra lambda is significant only 20 percent of the time. Full results are omitted for brevity. Thus, the evidence against CGARCH2L is weak and one should consider risk-return evidence from both 1- and 2-lambda models.

4.2 The CGARCH Model with One Lambda

Panel A of Table 1 shows that the CGARCH model (2) with daily data supports a positive risk-return relation. With a free constant term, $\hat{\lambda}$ is positive in 16 markets and also significant at the 10% level in six: Austria, Denmark, Germany, Italy, Spain, and the United States. All negative $\hat{\lambda}$s are insignificant.

[Place Table 1 about here]

We now investigate the relative contribution of daily data versus the CGARCH models in explaining the difference between our results and those of previous authors. Although full weekly results are omitted for brevity, CGARCH estimation on weekly data provides much less support for a positive risk-return relation than does estimation on daily data: $\hat{\lambda}$ is positive in only 10 markets and significantly positive in only two. This is consistent with Bali and Peng (2006), who find that 5-minute data more strongly support a positive risk-return relation than do daily data.

Simulations—calibrated to U.S. data—indicate that estimates from daily data have better properties than those from weekly data. Again, full results are omitted for brevity.

GARCH models offer slightly weaker evidence of a positive risk-return relation than do the CGARCH models (Panel B of Table 1). Most GARCH estimates of $\hat{\lambda}$ (Panel B) are smaller than CGARCH estimates (Panel A). $\hat{\lambda}$ is positive in 15 (16) countries in the GARCH (CGARCH) models. Finally, $\hat{\lambda}$ is significant at the 10% level in 5 (6) markets for the GARCH (CGARCH) model. While both daily data and the CGARCH model strengthen the case for a positive risk-return relation, the former contributes much more.

Why might daily data provide stronger support for a positive risk-return relation? First, daily data estimate volatility and the risk-return relation more precisely than weekly data.

Second, Bali and Peng (2006) suggest that daily volatility is closely related to illiquidity, which earns a positive premium. Third, Guo and Whitelaw (2006) argue that ignoring the hedge demand in Merton's ICAPM biases the estimated risk-return relation. Investment opportunities change slowly, at the business cycle frequency. Their effects on conditional returns will be almost constant at a daily frequency, allowing precise identification of the risk-return relation.

4.3 The Constant in the Return Equation

Panel A of Table 1 supports Engle and Lee's (1999) and Christoffersen et al.'s (2006) findings by showing that excluding the constant in CGARCH models strengthens evidence of a positive risk-return relation. When $c \equiv 0$, $\hat{\lambda}$ is significantly positive at the 10% level in 12 countries, compared with only 6 countries when the constant is free. The GARCH model produces very similar patterns in panel B of Table 1.

Lanne and Saikkonen (2006) argue that restricting the constant to zero raises the power of the test under the null. Our simulations confirm this intuitive finding. If, however, one excludes the constant when it does belong, one estimates a misspecified model and tests of $\hat{\lambda}$'s significance will tend to reject the (correct) null that $\hat{\lambda}$ equals zero. While correctly excluding the constant improves the power of the test, doing so incorrectly generates too many rejections of the null hypothesis of no risk-return relation. Because we do not know the true data generating process, long samples are necessary for reliable inference.

4.4 The CGARCH Model with Two Lambdas

Panel C of Table 1 displays CGARCH model results with different prices of long- and short-run risk. Including a constant increases the variability of $\hat{\lambda}_1$ and $\hat{\lambda}_2$ compared to the single $\hat{\lambda}$ case (panel A). The parameter on long-run volatility ($\hat{\lambda}_2$) is highly correlated with the constant in the return equation, making it difficult to precisely estimate these parameters.

When $c \equiv 0$, $\hat{\lambda}_2$ is generally positive and statistically significant while $\hat{\lambda}_1$ is insignificant, with mixed signs. This international evidence is consistent with Engle and Lee's finding that long-run volatility appears to determine expected U.S. stock returns much more than the short-run component.

The significance of $\hat{\lambda}_2$ could be spurious, however. Because q_t has a positive mean, a positive $\hat{\lambda}_2$ permits the mean error $(\bar{\varepsilon})$ to be zero, even when $c \equiv 0$. Thus, a significant $\hat{\lambda}_2$ does not necessarily reflect covariance with long-run volatility. $\hat{\lambda}_2$'s frequent insignificance and/or mixed signs in the presence of a constant supports the conclusion that it does not determine returns.

5. Conclusion

This paper investigates the risk-return relation in international stock markets. The international data strongly prefer the CGARCH model to GARCH. In contrast with previous evidence from weekly data, daily CGARCH results support a positive risk-return relation. While long-run volatility appears to importantly determine the conditional equity premium, the evidence might be spurious.

References

Adrian, T. and J. Rosenberg, 2006, Stock Returns and Volatility: Pricing the Long-Run and Short-Run Components of Market Risk, Unpublished Working Paper, Federal Reserve Bank of New York.

Bali, T. and L. Peng, 2006, Is there a Risk-Return Tradeoff? Evidence from High-Frequency Data, Journal of Applied Econometrics, Forthcoming.

Christoffersen, P., K. Jacobs, and Y. Wang, 2006, Option Valuation with Long-run and Short-run Volatility Components, Unpublished Working Paper, McGill University.

Engle, R. and G. Lee, 1999, A Long-Run and Short-Run Component Model of Stock Return Volatility, in Cointegration, Causality and Forecasting, Edited by R. Engle and H. White, Oxford University Press.

Glosten, L., R. Jagannathan, and D. Runkle, 1993, On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, Journal of Finance 48, 1779-1801.

Guo, H., R. Whitelaw, 2006. Uncovering the Risk-Return Relation in the Stock Market, Journal of Finance 61, 1399-1431.

Lanne, M. and P. Saikkonen, 2006, Why Is It So Difficult to Uncover the Risk-Return Tradeoff in Stock Returns?, Economics Letters 92, 118-125.

Li, Q., J. Yang, C. Hsiao, and Y. Chang, 2005, The relationship between stock returns and volatility in international stock markets, Journal of Empirical Finance 12, 650-665.

Merton, R., 1973, An Intertemporal Capital Asset Pricing Model, Econometrica 41, 867-887.

Theodossiou, P. and U. Lee, 1995, Relationship between Volatility and Expected Returns across International Stock Markets, Journal of Business Finance & Accounting 22, 289-300.

Table 1: Risk-Return Relation in Daily Data

Country	Panel A: CGARCH with 1 Lambda			Panel B: GARCH			Panel C: CGARCH with 2 Lambdas				
	With Constant		No Constant	With Constant		No Constant	With Constant			No Constant	
	$\hat{\lambda}$	$\hat{c}$	$\hat{\lambda}$	$\hat{\lambda}$	$\hat{c}$	$\hat{\lambda}$	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{c}$	$\hat{\lambda}_1$	$\hat{\lambda}_2$
Australia	-0.479	0.032*	2.435**	-1.421	0.036**	1.775*	8.907*	-5.614*	0.131**	-0.493	1.853
Austria	3.749**	-0.018***	1.541	3.415***	-0.017***	1.382	20.996***	10.883***	-0.012***	21.868***	10.717***
Belgium	2.560	0.008	3.452***	2.285	0.011	3.551***	9.756***	543.271***	-4.501***	2.283	3.517***
Canada	0.763	0.016	2.623**	1.132	0.012	2.467**	-62.093***	6.644***	0.014	-60.573***	8.583***
Denmark	3.823***	-0.025**	1.305	4.497***	-0.026***	1.652	3.581**	5.825	-0.043	3.949***	1.186
France	0.304	0.026	2.084**	0.294	0.024	1.950**	7.233*	-5.286	0.162**	0.330	1.687*
Germany	2.756***	0.006	3.195***	1.979*	0.012	2.792***	0.699	4.928*	-0.041	2.290	2.986***
Hong Kong	0.278	0.048**	1.595***	0.193	0.053***	1.583***	-0.338	0.673	0.023	-0.821	1.060*
Italy	2.510*	-0.035	0.635	2.795*	-0.039	0.729	13.686***	-13.825***	0.386***	4.547**	1.296
Japan	1.433	-0.006	1.014	0.948	0.002	1.079	-4.462*	5.322***	-0.094***	0.838	0.977
Netherlands	1.208	0.032**	3.602***	0.213	0.041***	3.127***	-0.397	2.066	0.007	-0.778	2.302*
Norway	-0.582	0.007	-0.232	-1.236	0.018	-0.289	-4.540**	6.686**	-0.149**	-2.304	-0.379
Singapore	0.494	0.012	1.115	-0.075	0.022*	1.067	2.111	-0.572	0.045	0.301	0.970
Spain	3.517***	-0.055***	-0.084	3.522***	-0.050***	-0.033	3.310**	6.409	-0.092	4.188***	-0.237
Sweden	-0.108	0.043***	2.340***	0.116	0.039**	2.351***	9.179***	-13.867***	0.381***	-0.014	1.938**
Switzerland	1.863	0.033***	4.909***	1.257	0.038***	4.523***	1.067	2.415	0.023	-0.531	3.796***
UK	0.459	0.025	2.344**	0.038	0.026	1.946**	0.093	0.870	0.016	-0.584	1.686
US	2.544*	0.007	3.124***	1.084	0.017	2.512***	-2.136	5.159*	-0.053	1.572	2.712**
World	0.499	0.026***	4.697***	-1.148	0.027***	3.175**	21.771***	-12.152**	0.170***	-1.529	3.109*
CRSP(VW)	1.288	0.042***	5.470***	0.593	0.043***	4.530***	5.010	-1.242	0.089	-1.308	3.719***

Notes: The table displays coefficient estimates from maximum likelihood estimation of GARCH, CGARCH and CGARCH with 2 lambda models, equations (1) through (3) in the text. Three, two and one asterisks indicate statistical significance at the 1-, 5- and 10-percent level, respectively.