Important Information
concerning the
Common Final Exam
in
CALCULUS II
15-MATH-252
The 99

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1 What are ***The 99***?

We are trying something new for the Common Final Exam in Calculus II that will be given in March, 2010. We have created a collection of 99 problems, known as ***The 99***. Approximately two-thirds of the questions on the common final will be a modification of one of these. There is more to Calculus II than is covered in ***The 99***, but if you master these questions you will have a solid foundation in Calculus II, which will be of great help in Calculus II,III, IV and Differential Equations.

2 What is the Point of ***The 99*** ?

The purpose is to give you a place to start studying for the Common Final; remember the bulk of the common final will consist of questions taken from ***The 99***.

3 Can I Ignore everything else?

Probably not a good idea. The questions from ***The 99*** cover $2/3^{\text{rd}}$ of $1/2$ of the final exam.

4 Where Can You Find ***The 99*** ?

- Most instructors have put 10 problem sets of 10 problems each[1] on your *CalcPortal*. & you can repeatedly practice the problems. Also you will discover what we mean by 'modify.' [2] The questions on the final will have the same sort of parameter changes.

- Your instructor may have put a version on Bb.

- On the door of office 606 B Old Chem. there will be an Errata sheet, answers and copies of ***The 99*** . There will be copies in the MLC and in the Mathematics Department Office 839 C OC.

[1]More precisely, put 9 problem sets of 10 problems each, and 1 problem set of 9 problems.

[2]The ultimate judge of what is an OK modification versus a non-0K modification is solely up to the Calculus committee.

Chapter 5: The Integral

§ 5.2: The Definite Integral

1. (§5.2,# 29) Calculate the Riemann sum $R(f, P, C)$ for

$$f(x) = x - 1, P = \{-2, -1.8, -1.5, 0\}, C = \{-1.9, -1.5, -0.75\}.$$

2. (§5.2,# 33)Calculate $\int_0^7 (5x+5)\,\mathrm{dx}$.

3. (§5.2,# 35) Calculate $\int_0^{10} (u^2+3u)\mathrm{du}$.

4. (§5.2,# 37) Calculate $\int_{-a}^{6} (x^2+x)\,\mathrm{dx}$.

5. (§5.2,#57) Assuming that $\int_0^1 f(x)\,\mathrm{dx} = 1$, $\int_0^2 \mathrm{f(x)}\,\mathrm{dx} = 4$ and $\int_1^4 \mathrm{f(x)}\,\mathrm{dx} = 7$. Now calculate $\int_2^4 f(x)\,\mathrm{dx}$.

§5.3: The Fundamental Theorem of Calculus: Part 1

6. (§5.3,#37) Find $\int_0^{\pi/4} \sec^2 t\,\mathrm{dt}$.

7. (§5.3,#45) Find $\int_{\pi/3}^{\pi} |\cos t|\,\mathrm{dt}$.

8. (§5.3,#51) Evaluate the integral in terms of the constant a. $\int_a^{9a} \frac{1}{x}\,\mathrm{dx}$.

§5.4: The Fundamental Theorem of Calculus: Part 2

9. (§5.4,#17) The antiderivative $F(x)$ of $f(x) = \sec x$ satisfying the initial condition $F(-2) = 0$ is given by $\int_a^b \sec t\,\mathrm{dt}$. Find a and b.

10. (§5.4,#19) Calculate the derivative. $\frac{\mathrm{d}}{\mathrm{dx}} \int_0^x (8t^2 - 4t)\,\mathrm{dt}$.

This is Problem Set 1.

11. (§5.4,#21) Find $\frac{\mathrm{d}}{\mathrm{dt}} \int_{60}^{t} \cos(7x)\,\mathrm{dx}$.

12. (§5.4,#27) Find $G'(x)$ where $G(x) = \int_{11^4}^{x^4} \tan t\,\mathrm{dt}$.

13. (§5.4,#31) Calculate the derivative. $\frac{\mathrm{d}}{\mathrm{ds}} \int_{0}^{\cos s} (3u^6 - 3u)\mathrm{du}$.

14. (§5.4,#33) Find $G'(x)$ where $G(x) = \int_{x^7}^{0} \sin^2 t\,\mathrm{dt}$.

§5.5: **The Integral as Rate**

15. (§5.5,#3) A population of insects increases at a rate of $280 + 8t + .0.6t^2$ insects/day. Find the insect population after 6 days assuming that there are 40 insects at $t = 0$

16. (§5.5,#13) The rate in liters per minute (l/min) at which water drains from a tank is recorded at half minute intervals in the table below. Use the average of the left and the right hand-endpoint approximations to estimate the total amount of water drained during the first 3 minutes.

t min	0	0.5	1	1.5	2	2.5	3
l/min	52	48	40	36	32	28	44

17. (§5.5,#33) The traffic flow rate past a certain point on a highway is $q(t) = 1200+2000t-420t^2$ where t is in hours and $t = 0$ is 8 AM. How many cars pass by during the time interval from 8 to 10 AM?

§5.6: **Substitution Method**

18. (§5.6,#33) Evaluate the indefinite integral. $\int (2x-1)^5\,\mathrm{dx}$.

19. (§5.6,#35) Evaluate the indefinite integral. $\int \frac{1}{\sqrt{8x-3}}\,\mathrm{dx}$.

20. (§5.6,#37) Evaluate the indefinite integral. $\int x\sqrt{x^2-5}\,\mathrm{dx}$.

This is Problem Set 2.

21. (§5.6,#39) Evaluate the indefinite integral. $\int \frac{1}{(x+5)^2}\,\mathrm{dx}$.

22. (§5.6,#43) Evaluate the indefinite integral. $\int \frac{6x^5+4x^3}{(x^6+x^4)^3}\,\mathrm{dx}$.

23. (§5.6,#47) Evaluate the indefinite integral. $\int x(x+1)^{1/5}\,\mathrm{dx}$.

24. (§5.6,#51) Evaluate the indefinite integral. $\int (\sin x)^5 \cos x\,\mathrm{dx}$.

25. (§5.6,#67) Evaluate the indefinite integral. $\int \frac{(\ln x)^5}{x}\,\mathrm{dx}$.

26. (§5.6,#69) Evaluate the indefinite integral. $\int \frac{\mathrm{dx}}{x\ln(5x)}\,\mathrm{dx}$.

27. (§5.6,#91) Evaluate the indefinite integral. $\int \tan^7 x \sec^2 x\,\mathrm{dx}$

§5.7: **Further Transcendental Functions**

28. (§5.7,#3) Evaluate the definite integral. $\int_1^3 \frac{\mathrm{dx}}{x}$.

29. (§5.7,#7) Evaluate the definite integral. $\int_0^{1/3} \frac{1}{\sqrt{1-x^2}}\,\mathrm{dx}$.

30. (§5.7,#13) Evaluate the definite integral. $\int_0^8 \frac{\mathrm{dx}}{x^2+16}$.

This is Problem Set 3.

31. (§5.7,#17) Evaluate the indefinite integral. $\int \frac{1}{\sqrt{25-9x^2}}\,\mathrm{dx}$.

32. (§5.7,#25) Evaluate the indefinite integral. $\int \frac{(\arctan x)^6}{1+x^2}\,\mathrm{dx}$.

33. (§5.7,#43) Evaluate the indefinite integral. $\int e^t\sqrt{e^t+3}\,\mathrm{dt}$.

34. (§5.7,#57) Evaluate the indefinite integral. $\int \tan(6x+1)\,\mathrm{dx}$.

§5.8: **Exponential Growth and Decay**

35. (§5.8,#9) Find the solution to $y' = 2y$ satisfying $y(3) = 7$, where $y = y(t)$.

Chapter 6: Applications of the Integral

§6.1: **Area Between Two Curves**

36. (§6.1,#5) Find the area between $y = \sin x$ and $y = \cos x$ over the interval $[0, \pi/4]$.

37. (§6.1,#11) Find the area between $y = e^x$ and $y = e^{4x}$ over the interval $[0, 1]$.

38. (§6.1,#19) Find the area between $y = x^3 - 6x + 3$ and $y = 11 - 3x^2$.

39. (§6.1,#41) Sketch the region enclosed by the curves $x - 3 = 2y$ and $x - 2 = (y-1)^2$, and compute its area.

This is Problem Set 4.

40. (§6.1,#45) Sketch the region enclosed by the curves $y = 3\sin x, y = 3\csc^2 x, x = \pi/4, x = (3\pi)/4$ and compute its area.

§6.2: **Setting Up Integrals: Volume, Density and Average Value**

41. (§6.2,#5) Find the volume of liquid needed to fill a sphere of radius R to height $R/3$.

42. (§6.2,#9) Find the volume of a solid whose base is the circle $x^2 + y^2 = 9$ and the cross sections perpendicular to the x-axis are triangles whose height and base are equal.

43. (§6.2,#25) Find the total mass of a 5-meter rod whose linear density function is $\rho(x) = 1 + 0.5\sin(\pi x)$ kg/m for $0 \leq x \leq 5$.

§6.3: **Volumes of Revolution**

44. (§6.3,#5) Find the volume of the solid obtained by rotating the region under the graph of the function $f(x) = x^2 - 3x$ about the x-axis over the interval $[0, 4]$.

45. (§6.3,#9) Find the volume of the solid obtained by rotating the region under the graph of the function $f(x) = \dfrac{2}{x+1}$ about the x-axis over the interval $[0, 5]$.

46. (§6.3,#11) Find the volume of the solid obtained by rotating the region under the graph of the function $f(x) = e^x$ about the x-axis over the interval $[0, 4]$.

47. (§6.3,#21) Find the volume of the solid obtained by rotating the region enclosed by the graphs of the functions $y = x^4$ and $y = \sqrt[4]{x}$ over the interval $[0, 1]$.

48. (§6.3,#39) Find the volume of the solid obtained by rotating the region enclosed by the graphs of the functions $y = \frac{9}{x^2}$ and $y = 10 - x^2$ about $y = -1$.

§6.4: **The Method of Cylindrical Shells**

49. (§6.4,#11) Use the Shell Method to compute the volume of the solids obtained by rotating the region enclosed by the graphs of the functions $y = x^2$, $y = 8 - x^2$ & $x = 1$ about the y-axis.

This is Problem Set 5.

50. (§6.4,#13) Use the Shell Method to compute the volume of the solids obtained by rotating the region enclosed by the graphs of the functions $y = x^6$, $y = \sqrt[6]{x}$ about the y-axis .

51. (§6.4,#23) Use the Shell Method to compute the volume of rotation about the x-axis for the region underneath the graph of $y = x$ where $0 \leq x \leq 3$.

52. (§6.4,#39) Use the Shell Method to compute the volume of the solids obtained by rotating the region below the graph of the functions $f(x) = x^2$, and above $y = 0$ about the y-axis .

53. (§6.4,#45) Find the volume of the solid obtained by rotating the region enclosed by the graphs of $y = e^{-x}$ and $y = 1 - e^{-x}$ and $x = 0$ about $y = 2.5$.

§6.5: **Work and Energy**

54. (§6.5,#3) Compute the work required to stretch a spring from equilibrium to 7 cm past equilibrium, assuming the spring constant is $k = 130$ kg/s^2.

55. (§6.5,#7) If 5 J of work are needed to stretch a spring t0 10 cm beyond equilibrium, how much work is required to stretch it 19 cm beyond equilibrium?

56. (§6.5,#11) Calculate the work against gravity required to build tower of height 16 ft and square base of side 10 ft out of brick . Assume the density of brick is 70 lbs/ft^3.

Chapter 7: Techniques of Integration

§7.2: **Integration by Parts**

57. (§7.2,#1) Evaluate the integral using the Integration by Parts formula with the given choice of u and v'. $\int 4x \sin(5x)\,\mathrm{dx}$, $u = x$ and $v' = \sin 5x$.

58. (§7.2,#3) Evaluate the integral using the Integration by Parts formula with the given choice of u and v'. $\int (4x + 7)e^{5x}\,\mathrm{dx}$, $u = 4x + 7$ and $v' = e^{5x}$.

59. (§7.2,#5) Evaluate the integral using the Integration by Parts formula with the given choice of u and v'. $\int x^{13} \ln x\,\mathrm{dx}$, $u = \ln x$ and $v' = x^{13}$.

This is Problem Set 6.

60. (§7.2,#11) Use Integration by Parts to evaluate the integral. $\int 7x\cos(5x)\,\mathrm{dx}$.

61. (§7.2,#15) Use Integration by Parts to evaluate the integral. $\int 4x^2 \ln x\,\mathrm{dx}$.

62. (§7.2,#21) Use Integration by Parts to evaluate the integral. $\int 3x6^x\,\mathrm{dx}$.

63. (§7.2,#45) Compute the definite integral. $\int_0^3 xe^{5x}\,\mathrm{dx}$.

64. (§7.2,#47) Compute the definite integral. $\int_0^{13} x\sqrt{13-x}\,\mathrm{dx}$.

65. (§7.2,#49) Compute the definite integral. $\int_1^{21} \sqrt{x}\ln x\,\mathrm{dx}$.

§7.3: **Trigonometric Integrals**

66. (§7.3,#3) Use the method of odd powers to evaluate the integral. $\int \sin^3 x\cos^8 x\,\mathrm{dx}$.

67. (§7.3,#5) Use the method of odd powers to evaluate the integral. $\int \sin^3 x\cos^8 x\,\mathrm{dx}$.

68. (§7.3,#15) Evaluate the integral using reduction formulas as necessary. $\int \tan^5 x\sec^2 x\,\mathrm{dx}$.

§7.4: **Trigonometric Substitution**

69. (§7.4,#5) Use the indicated substitution to evaluate the integral. $\int \sqrt{49-x^2}\,\mathrm{dx}, \mathrm{x} = 7\sin \mathrm{t}$.

This is Problem Set 7.

70. (§7.4,#7) Use the indicated substitution to evaluate the integral. $\int \frac{1}{x\sqrt{x^2-36}}\,\mathrm{dx}, \mathrm{x} = 6\sec\mathrm{t}$.

71. (§7.4,#13) Evaluate the integral using trigonometric substitution. $\int \frac{x^2\,\mathrm{dx}}{\sqrt{36-x^2}}$.

72. (§7.4,#15) Evaluate the integral using trigonometric substitution. $\int \sqrt{99+9x^2}\,\mathrm{dx}$.

73. (§7.4,#19) Evaluate the integral using trigonometric substitution. $\int \frac{1}{x^2\sqrt{7-x^2}}\,\mathrm{dx}$.

74. (§7.4,#29) Evaluate the integral using trigonometric subsitution. $\int x^3\sqrt{4-x^2}\,\mathrm{dx}$.

§7.6: **The Method of Partial Fractions**

75. (§7.6,#1) Find the partial fraction decomposition for the rational function: $\frac{8x^2-7x+344}{(x^2+36)(x-8)}$.

76. (§7.6,#9) Evaluate the integral. $\int \frac{\mathrm{dx}}{(x-2)(x-11)}$.

77. (§7.6,#17) Evaluate the integral. $\int \frac{3x-7}{(x-1)(x-1)^2}\,\mathrm{dx}$.

78. (§7.6,#21) Evaluate the integral. $\int \frac{4x^2+31x+48}{x(x+4)^2}\,\mathrm{dx}$.

79. (§7.6,#31) Evaluate the integral. $\int \frac{x^2\,\mathrm{dx}}{81x^2+36}$.

This is Problem Set 8.

§7.7: **Improper Integrals**

80. (§7.7,#15) Determine whether the improper integral converges and, if so, evaluate it. $\displaystyle\int_{\infty}^{-2} \frac{1}{(x+3)^{3/2}}\,\mathrm{dx}$.

81. (§7.7,#21) Determine whether the improper integral converges and, if so, evaluate it. $\displaystyle\int_{-\infty}^{0} e^{3x}\,\mathrm{dx}$.

82. (§7.7,#27) Determine whether the improper integral converges and, if so, evaluate it. $\displaystyle\int_{0}^{\infty} \frac{7\,\mathrm{dx}}{1+x}\,\mathrm{dx}$.

83. (§7.7,#43) Determine whether the improper integral converges and, if so, evaluate it. $\displaystyle\int_{1}^{0} 4x\ln x\,\mathrm{dx}$.

84. (§7.7,#65) Use the comparison test to determine weather the improper integral converges or not. $\displaystyle\int_{0}^{\infty} \frac{\mathrm{dx}}{\sqrt[3]{x^4+4}}$.

Chapter 8: Further Applications of the Integral and Taylor Polynomials

§8.1: **Arc Length and Surface Area**

85. (§8.1,#5) Calculate the arc length of $y = 3x + 1$ over the interval $[0, 8]$.

86. (§8.1,#7) Calculate the arc length of $y = x^{3/2}$ over the interval $[2, 3]$.

87. (§8.1,#9) Calculate the arc length of $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}$ over the interval $[1, 3e]$.

88. (§8.1,#33) Compute the surface area of revolution of $y = 4x + 3$ about the x-axis over the interval $[3, 4]$.

89. (§8.1,#37) Compute the surface area of revolution of $y = e^x$ about the x-axis over the interval $[2, 3]$.

This is Problem Set 9.

90. (§8.1,#39) Compute the surface area of revolution of $y = \sin x$ about the x-axis over the interval $[0, 3\pi]$.

§8.3: **Center of Mass**

91. (§8.3,#11) Find the centroid of the region lying underneath the graph of the function $f(x) = \sqrt{x}$ over the interval $[0, 14]$.

92. (§8.3,#13) Find the centroid of the region lying underneath the graph of the function $f(x) = 9 - x^2$ over the interval $[0, 3]$.

93. (§8.3,#15) Find the centroid of the region lying underneath the graph of the function $f(x) = e^{-x}$ over the interval $[0, 3]$.

94. (§8.3,#17) Find the centroid of the region lying underneath the graph of the function $f(x) = \sin x$ over the interval $[0, \pi/5]$.

§8.4: **Taylor Polynomials**

95. (§8.4,#1) Calculate the Taylor polynomials $T_2(x)$ and $T_3(x)$ centered at $a = 0$ if $f(x) = \sin x$.

96. (§8.4,#5) Calculate the Taylor polynomials $T_2(x)$ and $T_3(x)$ centered at $a = 0$ if $f(x) = \tan x$.

97. (§8.4,#7) Calculate the Taylor polynomials $T_2(x)$ and $T_3(x)$ centered at $a = 0$ if $f(x) = \dfrac{1}{x^2 + 1}$.

98. (§8.4,#23) Calculate the Taylor polynomials $T_2(x)$ and $T_3(x)$ centered at $a = 1$ if $f(x) = e^x$.

99. (§8.4,#23) Calculate the Taylor polynomials $T_2(x)$ and $T_3(x)$ centered at $a = 1$ if $f(x) = \sqrt{x}$.

This is Problem Set 10.