

Exercise: For each real number $a > 0$, find the value of the series $\sum_{n=-\infty}^{\infty} \frac{\sin(2\pi an)}{n}$,

and also the series $\sum_{n=1}^{\infty} \frac{\sin(2\pi an)}{n}$. Express your answers as a function of a .

It should be understood that the $n = 0$ term of the first series has a value of $2\pi a$.

Hint:

You will probably find it helpful to work with the function $f(x) = \begin{cases} 1, & \text{if } |x| < a \\ \frac{1}{2}, & \text{if } x = \pm a \\ 0, & \text{if } |x| > a \end{cases}$