Math 416, Supplementary Exercise

1) Prove that $\displaystyle\int_{\mathbb{R}} \left(\int_{\mathbb{R}} \frac{x^2 - y^2}{(1 + x^2 + y^2)^2} \, dx \right) dy \neq \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \frac{x^2 - y^2}{(1 + x^2 + y^2)^2} \, dy \right) dx.$

Computing the values exactly is one form of proof.

Explain why Fubini's Theorem does not apply to these integrals.