Name: ____ April 5, 2006

Section Number/Time ____________________________

TA ____________________________

MATH 302, MIDTERM #2

Directions: This is a pencil-and-paper exam. You are asked to put away all books, notes, calculators, cell phones, and other computing and/or telecommunications equipment. The last page of this booklet is blank and is intended for use as scrap paper. Additional sheets of paper are available upon request.

Grading: Each numbered exam question is worth 5 points, for a total of 50 points.
There is a two-point extra credit question at the end of Problem 10.

Special Note: Many of the problems in this exam are interrelated. If the answer to one question appears to require the answer to a previous question which you have not solved, you may instead explain how this missing information would be used to solve the problem.

1. Find all solutions (i.e. the general solution) to the differential equation $y''(t) + y'(t) - 6y(t) = 0$.

2. Find all solutions to the differential equation $y''(t) + y'(t) - 6y(t) = 10e^{2t}$.

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3. Find the solution to the equation $y''(t) + y'(t) - 6y(t) = 10e^{2t}$ which satisfies the initial conditions $\begin{cases} y(0) = -1 \\ y'(0) = 0 \end{cases}$

4a. [3 pts.] Describe what happens to this solution $y(t)$ in the limit as $t \to \infty$ and in the limit $t \to -\infty$.

4b. [2 pts.] If we had chosen a different set of inital conditions in Problem 3, how could that change the limiting behavior you just described?

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5. A mass hanging on a (damped) spring will move according to the equation $my''(t) + \gamma y'(t) + ky(t) = 0$.
Suppose we are given a spring with a fixed elasticity constant k and damping constant γ,
and we are allowed to choose what size mass is attached.

For what values of $m > 0$ will the mass oscillate infinitely many times as it approaches the rest position?
[In other words, for which $m > 0$ is this an under-damped system?]

6. Suppose that the vector functions $\vec{x}^{(1)}(t) = \begin{bmatrix} 1 \\ \sin(2t) \\ 0 \end{bmatrix}$, $\vec{x}^{(2)}(t) = \begin{bmatrix} \sin(2t) \\ 1 \\ \cos(2t) \end{bmatrix}$, $\vec{x}^{(3)}(t) = \begin{bmatrix} 0 \\ \cos(2t) \\ 1 \end{bmatrix}$

are all solutions of a linear system of equations $\vec{x}'(t) = P(t)\vec{x}(t)$,
with $P(t)$ being a 3×3 matrix of continuous functions $p_{ij}(t)$.

What is the Wronskian $W[\vec{x}^{(1)}, \vec{x}^{(2)}, \vec{x}^{(3)}](t)$? Is this a fundamental set of solutions?

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Problems 7-9 will all involve the matrix $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ in some way.

7. Calculate the determinant $\det(A - rI)$ and find the eigenvalues of A.

8. The eigenvectors of A are (in no particular order): $\xi^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\xi^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Write the general solution for the system of equations $\vec{x}' = A\vec{x}$.

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9. Find the solution to the system of equations $\vec{x}' = A\vec{x}$ which satisfies the initial conditions $\vec{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.
What happens to this solution in the limit $t \to \infty$?

10. The linear system of equations $\vec{x}'(t) = \begin{bmatrix} \alpha & -2 \\ 2 & 1 \end{bmatrix} \vec{x}$ can exhibit a wide variety of phase portraits depending on the value of α.
What kind of equilibrium point is there at the origin if $\alpha = 6$?
What kind of equilibrium point is there at the origin if $\alpha = -2$?

11. [Extra credit] Is the origin ever a saddle point? If so, for what range of α does this occur?
Show your work on the next page.