

Name: _____

February 22, 2006

Section Number/Time _____

TA _____

MATH 302, MIDTERM #1

Directions: This is a pencil-and-paper exam. You are asked to put away all books, notes, calculators, cell phones, and other computing and/or telecommunications equipment. The last page of this booklet is blank and is intended for use as scrap paper. Additional sheets of paper are available upon request.

Each numbered exam question is worth 10 points, for a total of 50 points. In questions that are subdivided into several parts, each part is worth an approximately equal share of 10 points. There is an additional 3-point extra credit question at the end of problem 5.

1. Problems 1-3 will deal with the following list of first-order differential equations:

a) $\frac{dy}{dx} = 2(xy^2 + x)e^{x^2}$

b) $\frac{dy}{dx} = 2xy + 3x^2e^{x^2}$

c) $2ye^{x^2} \frac{dy}{dx} = 4x^3 - 2xy^2e^{x^2}$

Classify each of the above equations as being linear, separable, exact, or none of these.

Name _____

Section/TA _____

2. Choose one of the differential equations in Problem 1.
Find the solution which passes through the point $(0, 1)$.
Express your answer in the form $y = [\text{a function of } (x)]$.

Name _____

Section/TA _____

3. Choose another one of the differential equations in Problem 1.

Find the solution which passes through the point $(0, 1)$.

Express your answer in the form $y = [\text{a function of } (x)]$.

Name _____

Section/TA _____

4. Consider the autonomous equation $y'(t) = \sin(y(t))$.

a) [3 points] What are the equilibrium (constant) solutions for this equation?

b) [4 points] Which of these are the stable equilibrium solutions, and which are unstable?

c) [3 points] Suppose $y(t)$ is a solution with the initial value $y(0) = 10$. What is $\lim_{t \rightarrow \infty} y(t)$?

Name _____

Section/TA _____

5. Newton's Law of Cooling states that an object whose temperature is different from the surrounding environment will heat up (or cool down) at a rate proportional to the difference

[*temperature of environment* – *temperature of object*].

- a) [4 points] Write an equation describing the temperature of an object placed in a 160°C oven. Use the proportionality constant $k = 2$, measured in units of [1/hours].

- b) [6 points] If you place a pot of 25°C water in the oven, how long does it take to reach a boil (100°C)?

Extra Credit) What temperature oven would have brought the water to a boil in exactly half the time?
Start your work here, then continue on the scrap paper or the back of this page.