TEM Waves

Characteristic impedance can be written as

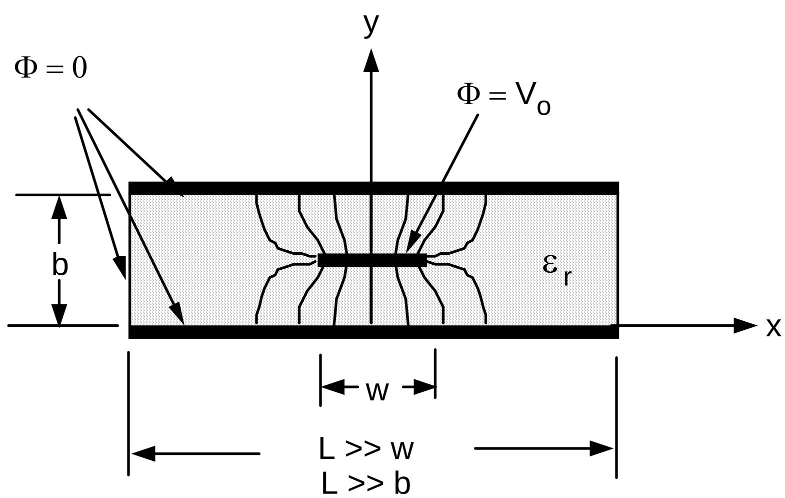
$$Z_o = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_r}}{cC}$$

where

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o \varepsilon_r \varepsilon_o}} = \frac{c}{\sqrt{\varepsilon_r}}$$

Therefore, if one can find the capacitance of the transmission line without and without the dielectric, the effective dielectric constant and the characteristic impedance of the line can be can be found.

Example 1: **Stripline transmission Line**.



- Solve Laplace's equation

$\nabla_t^2 \Phi(x,y) = 0$ in the two regions Region I $\rightarrow$ $0 \le y \le b/2$ and
Region II $\rightarrow$ $b/2 \le y \le b$

- Find eigensolutions $\Phi_n(x,y)$ and write for the two regions solutions in terms of infinite sum of eigensolutions

$$\Phi_I = \sum_{1}^{\infty} A_n \Phi_n(x,y) \quad \text{and } \Phi_2 = \sum_{1}^{m} B_m \Phi_m(x,y)$$

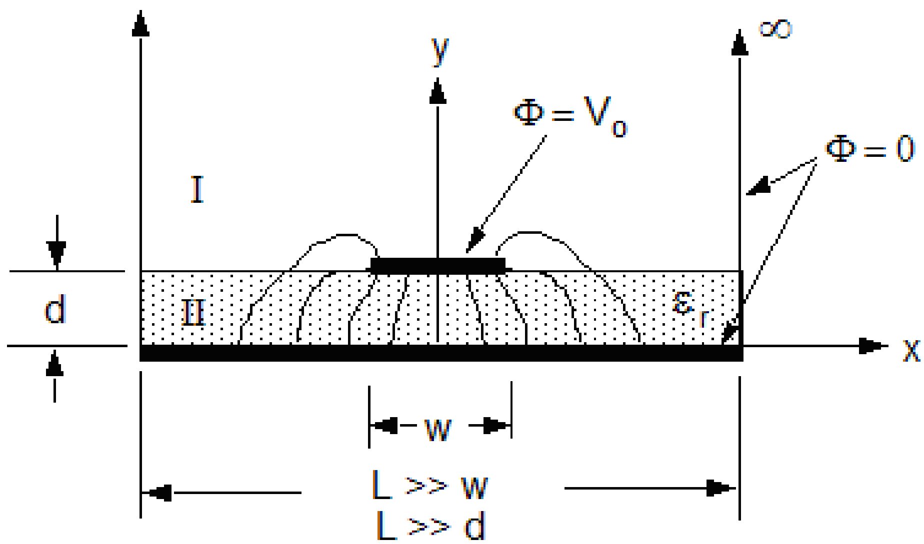
- Find $E_y = -\nabla_t \Phi(x,y)$ in each region

- Find the surface charge density

$$\rho_s = D_{yII}(x, y = (b/2)^+) - D_{yI}(x, y = (b/2)^-)$$

- Assume $\rho_s = const.$ or find a self-consistent ρ_s
- Find $V = -\int_0^{b/2} \vec{E} \cdot d\vec{l}$ and $Q_s = \int_{-w/2}^{+w/2} \rho_s dx$
- Then $C = \dfrac{Q_s}{V}$ and $Z_o = \dfrac{\sqrt{\varepsilon_r}}{cC}$

Example 2. Microstrip-line



- Assume the potential goes to zero at $y=\infty$ and $|x|=L/2$
- Solve Laplace's equation

$$\nabla_t^2 \Phi(x,y) = 0$$ in the two regions Region I $\rightarrow$ $0 \le y \le d$ and Region II $\rightarrow$ $d \le y \le \infty$

- Find eigensolutions $\Phi_n(x,y)$ and write for the two regions solutions in terms of infinite sum of eigensolutions

$$\Phi_I = \sum_1^{\infty} A_n \Phi_n(x,y) \quad \text{and} \quad \Phi_2 = \sum_1^{m} B_m \Phi_m(x,y)$$

- Solve for A_n and B_m.

Note that $\Phi_1(x, b/2) = \Phi_{II}(x, b/2)$ for $w/2 \le |x| \le L/2$

- Find $E_y = -\dfrac{\partial}{\partial y}\Phi(x,y)$ and $E_x = -\dfrac{\partial}{\partial x}\Phi(x,y)$ in each region
- Find the surface charge density on the strip

$$\rho_s = D_{yII}(x, y = (b/2)^+) - D_{yI}(x, y = (b/2)^-)$$

- Assume $\rho_s = const.$ or find a self-consistent ρ_s
- Find $V = -\int_0^{b/2} \vec{E} \cdot d\vec{l}$ and the total $Q_s = \int_{-w/2}^{+w/2} \rho_s dx$
- Capacitance is calculated from $C = \dfrac{Q_s}{V}$
- These calculations are repeated twice:

(1) for the line without any dielectric which results in C_o

(2) for the line with a dielectric which results in C

- Then the effective dielectric constant is $\varepsilon_{eff} = \dfrac{C}{C_o}$ and
- finally $Z_o = \dfrac{\sqrt{\varepsilon_{eff}}}{cC}$