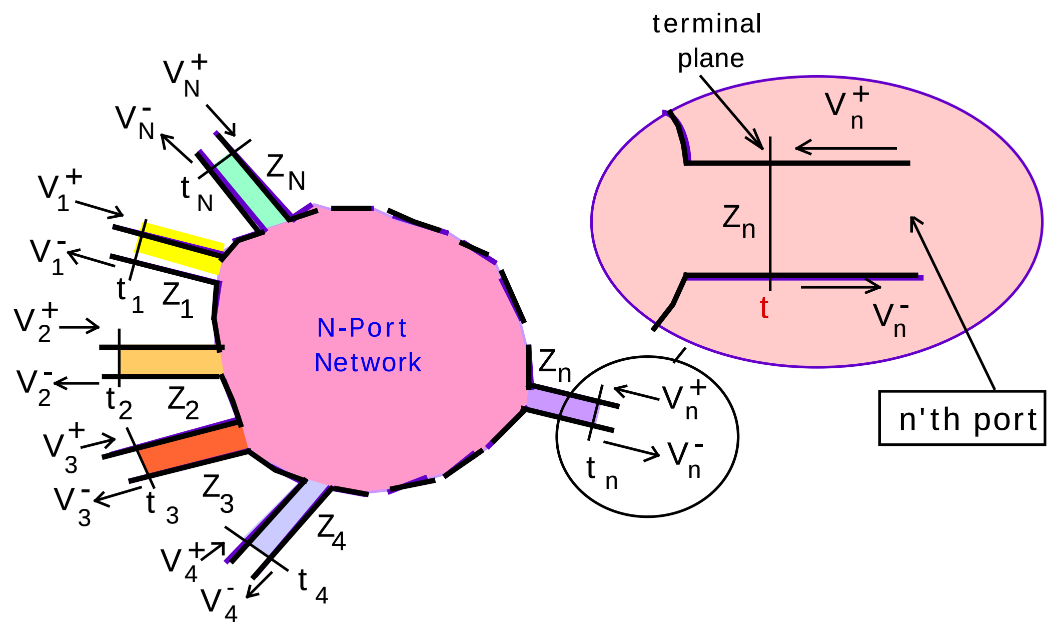
EE-611
Microwave Communications
Supplementary Notes.
Network Representation of Microwave Systems

S-Parameters

It is difficult or impossible to measure true voltages and currents at a given port. Therefore, impedance and admittance matrix concepts are not useful at microwave frequencies. In place of this, one can relate the incident voltages to reflected voltages at the ports. This results in a realizable and measurable characterization of the microwave circuits. In the N-port microwave system shown below, at each port and at the specified reference terminal plane, V_n^+, the incident and V_n^-, reflected wave are given. Each port has a characteristic impedance of Z_n ohms.



S-parameters are defined by

$$\begin{bmatrix} V_1^- \\ V_2^- \\ . \\ . \\ . \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & . & . & . & S_{1N} \\ S_{21} & S_{22} & . & . & . & S_{2N} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ S_{N1} & S_{N2} & . & . & . & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ . \\ . \\ . \\ V_N^+ \end{bmatrix}$$

Various S-parameters can be found from

$$S_{ii} = \left.\frac{V_i^-}{V_i^+}\right|_{\text{all } V_{n\neq i}^+ = 0}$$

i.e., all ports are matched to their characteristic impedance: all the reflected voltages are zero. S_{ii} in nothing else than the reflection coefficient of the port i at the terminal plane t_i.

Similarly,

$$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{\text{all } V_{n\neq j}^+ = 0}$$

This in nothing else than the transmission coefficient from port j to port i when all n ports (except n=j) are matched to their characteristic impedance.

Properties of S-Parameters

S parameter equation can be written in terms of matrices as

$$[V^-] = [S][V^+]$$

Here [S] matrix is given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & . & . & . & . & S_{1N} \\ S_{21} & S_{22} & . & . & . & . & S_{2N} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ S_{N1} & S_{N2} & . & . & . & . & S_{NN} \end{bmatrix}$$

Properties

- If the reference terminal planes are shifted toward outside by an electrical length βl, then the new [S'] matrix is given by

$$[S'] = \begin{bmatrix} e^{-j\beta_1 l_1} & 0 & 0 & . & . & . & 0 \\ 0 & e^{-j\beta_2 l_2} & 0 & . & . & . & 0 \\ 0 & 0 & e^{-j\beta_3 l_3} & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & . & . & . & e^{-j\beta_N l_N} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} & . & . & . & . & S_{1N} \\ S_{21} & S_{22} & . & . & . & . & S_{2N} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ S_{N1} & S_{N2} & . & . & . & . & S_{NN} \end{bmatrix} \begin{bmatrix} e^{-j\beta_1 l_1} & 0 & 0 & . & . & . & 0 \\ 0 & e^{-j\beta_2 l_2} & 0 & . & . & . & 0 \\ 0 & 0 & e^{-j\beta_3 l_3} & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & . & . & . & e^{-j\beta_N l_N} \end{bmatrix}$$

· If normalized voltages are used, the scattering matrix is symmetric, i.e., $S_{nm}=S_{mn}$. In matrix form this can be written as

$$[S] = [S]_t$$

· For a lossless network,

$$\sum_{n=1}^{N} |S_{ni}| = \sum_{n=1}^{N} S_{ni} S_{ni}^{*} = 1$$

$$\sum_{n=1}^{N} S_{ns} S_{nr}^{*} = 0 \qquad\qquad s \neq r$$

This implies that [S] is unitary, i.e.,

$$[S]^{*} = [S]_t^{-1}$$

· Normalized voltages are defined as

$$b_1 = \frac{V_1^-}{\sqrt{Z_1}} \qquad b_2 = \frac{V_2^-}{\sqrt{Z_2}} \qquad .... \qquad b_n = \frac{V_n^-}{\sqrt{Z_n}}$$

$$a_1 = \frac{V_1^+}{\sqrt{Z_1}} \qquad a_2 = \frac{V_2^+}{\sqrt{Z_2}} \qquad ..... \qquad a_n = \frac{V_n^+}{\sqrt{Z_n}}$$

· For a lossless network the total power is conserved. That is the incident power is equal to the reflected power

$$\sum_{i=1}^{N} |b_i|^2 = \sum_{n=1}^{N} |a_i|^2$$

· The generalized scattering parameters are then defined in terms of the normalized voltages (as an example for a two port network)

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

· The relation between the unnormalized and generalized (normalized) S parameters can be written as (as an example S_{12})

$$S'_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{\dfrac{V_1^-}{\sqrt{Z_1}}}{\dfrac{V_2^+}{\sqrt{Z_2}}} = \frac{\sqrt{Z_2}}{\sqrt{Z_1}} S_{12} = \sqrt{\frac{Y_1}{Y_2}}\, S_{12}$$

Note that S_{12} is equal to S_{12}' if the port impedances are the same.