ECECS-611
Microwave Communications

Transmission Lines as Resonant structures.

Consider a lossy transmission line with the following parameters.

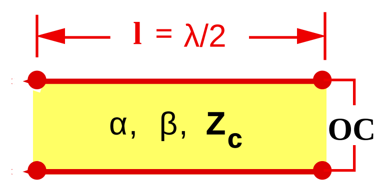
$\gamma=\alpha+j\beta$ with
β= propagation constant,
α= attenuation constant
Z_c= characteristic impedance.

For a lossy line, the input impedance can be calculated from

$$Z_{in} = Z_c \frac{Z_L + Z_c \tanh(\alpha+j\beta)}{Z_c + Z_L \tanh(\alpha+j\beta)}$$

Open circuited $\lambda/2$ line

An open circuited transmission line with a length $\lambda/2$ is shown in the figure below.



for Z_L=oo.

$$Z_{in} = Z_c \coth(\alpha+j\beta)$$

Using the identity

$$Z_{in} = Z_c \frac{1 + j\tan(\beta l)\tanh(\alpha l)}{\tanh(\alpha l) + j\tan(\beta l)}$$

for small losses

$$\tanh(\alpha l) \approx \alpha l$$

also

$$\tan(\beta l) = \tan(\omega\sqrt{\mu\varepsilon}\, l) = \tan[(\omega_o+\Delta\omega)\sqrt{\mu\varepsilon}\, l]$$

$$= \tan[\,\omega_o\sqrt{\mu\varepsilon}\, l + \Delta\omega\sqrt{\mu\varepsilon}\, l]$$

$$= \tan\left[\frac{\omega_o}{v}\, l + \Delta\omega\sqrt{\mu\varepsilon}\, l\right]$$

$$= \tan\left[\frac{2\pi}{\lambda}\, l + \Delta\omega\sqrt{\mu\varepsilon}\, l\right]$$

$$= \tan\left[\frac{2\pi}{\lambda}\,\frac{\lambda}{2} + \Delta\omega\sqrt{\mu\varepsilon}\, l\right]$$

$$= \tan[\pi + \Delta\omega\sqrt{\mu\varepsilon}\, l] = \tan(\,\Delta\omega\sqrt{\mu\varepsilon}\, l) \approx \Delta\omega\sqrt{\mu\varepsilon}\, l\,\frac{\omega_o}{\omega_o} = \frac{\Delta\omega}{\omega_o}\,\pi$$

Substituting these into Z_{in}

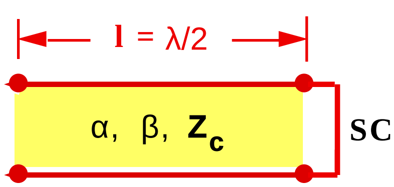
$$Z_{in} = Z_c\,\frac{1 + j\dfrac{\Delta\omega}{\omega_o}\,\pi\,(\alpha l)}{\alpha l + j\dfrac{\Delta\omega}{\omega_o}\,\pi} \approx \frac{Z_c}{\alpha l + j\dfrac{\Delta\omega}{\omega_o}\,\pi} = \frac{Z_c}{\alpha l}\left(\frac{1}{1 + j\dfrac{\Delta\omega}{\omega_o}\,\pi\,\dfrac{1}{\alpha l}}\right)$$

Therefore, this frequency variation is similar to a parallel RLC circuit. Comparing with the parallel RLC circuit elements, we can define the equivalent elements for the open-circuited line

$$R = \frac{Z_c}{\alpha l} \qquad C = \frac{\pi}{2\, Z_c\, \omega_o} \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

Short Circuited λ/2 line.

Consider now a short-circuited line of length $\lambda/2$. For this case, $Z_L=0$.



$$Z_{in} = Z_c \tanh[(\alpha + j\beta)l]$$

$$Z_{in} = Z_c\,\frac{\tanh(\alpha l) + j\tan(\beta l)}{1 + j\tan(\beta l)\tanh(\alpha l)}$$

$$\tan(\beta l) = \tan(\omega\sqrt{\mu\varepsilon}\; l) = \tan\left((\omega_o + \Delta\omega)\sqrt{\mu\varepsilon}\; l\right)$$

$$= \tan(\omega_o\sqrt{\pi\varepsilon}\, l + \Delta\omega\sqrt{\mu\varepsilon}\; l)$$

$$= \tan\left(\frac{2\pi}{\lambda_o}\frac{\lambda}{2} + \frac{\omega_o}{\omega_o}\Delta\omega\sqrt{\mu\varepsilon}\; l\right)$$

$$= \tan\left(\frac{2\pi}{\lambda_o}\frac{\lambda}{2} + \frac{2\pi}{\lambda_o}\frac{\lambda_o}{2}\frac{\Delta\omega}{\omega_o}\right)$$

$$= \tan\left(\pi + \pi\frac{\Delta\omega}{\omega_o}\right) \approx \pi\frac{\Delta\omega}{\omega_o}$$

For this case

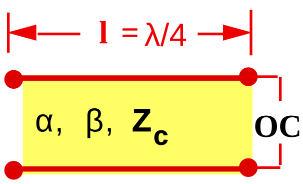
$$Z_{in} \approx Z_c \frac{\alpha l + j\left(\pi\frac{\Delta\omega}{\omega_o}\right)}{1 + j\left(\pi\frac{\Delta\omega}{\omega_o}\right)\alpha l} \approx Z_c\left(\alpha l + j\,\pi\frac{\Delta\omega}{\omega_o}\right)$$

Frequency variation of this circuits is similar to the series RLC circuit. Thus the equivalent circuit parameters can be written as

$$R = Z_c\,\alpha l \qquad L = \frac{\pi Z_c}{2\,\omega_o} \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

Similar expressions can be driven for

Open-Circuited line of λ/4 (Eq. to series LRC)



short-circuited Line of λ/4 (Eq. to parallel LRC)