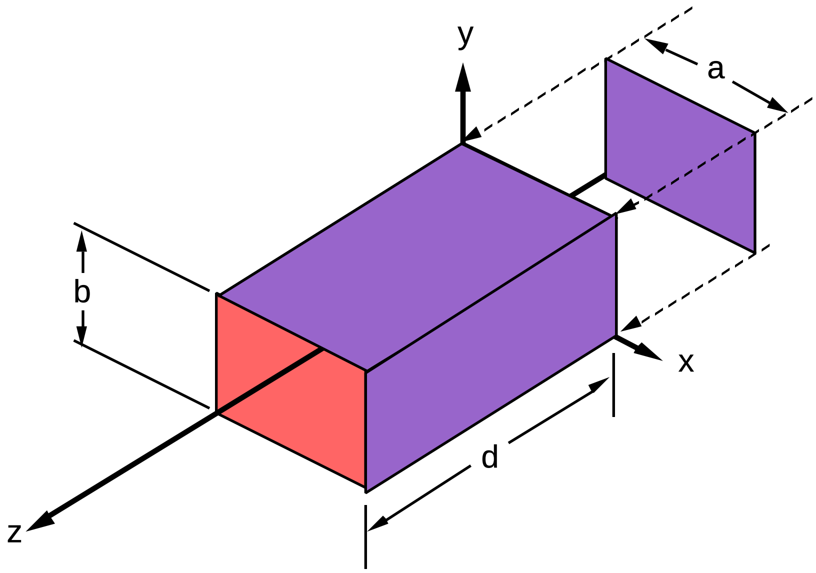
EE-611
Microwave Communications

Rectangular Cavity Resonators



Given a rectangular waveguide of dimensions (a,b). Assume a TE_{n0} mode is propagating in the waveguide. The total field is the sum of the positively and negatively traveling waves.

$$\mathbf{E}_y = E_o \sin\left(\frac{n\pi x}{a}\right) e^{-j\beta z} + \Gamma E_o \sin\left(\frac{n\pi x}{a}\right) e^{j\beta z}$$

Now a metal plate is attached to the waveguide at z=0 as shown in the figure. Since E_y is tangent to this metal at z=0, from the boundary conditions, $E_y(z=0)=0$

$$\mathbf{E}_y = E_o \sin\left(\frac{n\pi x}{a}\right) + \Gamma E_o \sin\left(\frac{n\pi x}{a}\right) = 0$$

$$\mathbf{E}_y = E_o \sin\left(\frac{n\pi x}{a}\right)(1+\Gamma) = 0 \qquad \rightarrow \quad \Gamma = -1$$

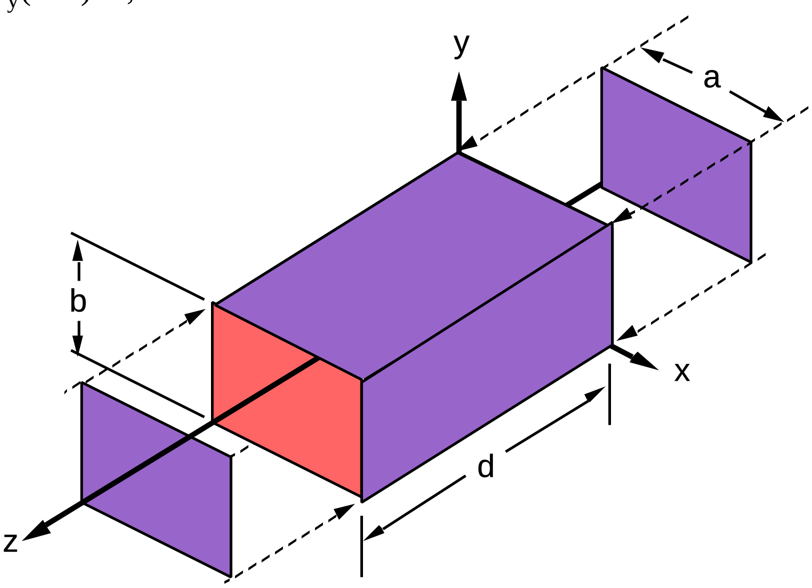
Which is expected. The wave is completely reflected back.
Substituting into the total E field, and simplifying the resulting expression

$$\mathbf{E}_y = E_o \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} + k\,\Gamma E_o \sin\left(\frac{\pi x}{a}\right) e^{j\beta z}$$

$$\mathbf{E}_y = E_o \sin\left(\frac{\pi x}{a}\right)\left(e^{-j\beta z} - e^{j\beta z} \right)$$

$$\mathbf{E}_y = -2jE_o \sin\left(\frac{\pi x}{a}\right)\sin(\beta z)$$

We now attached another metal plate at z=d. Again applying the boundary condition that $E_y(z=d)=0$,



$$\mathbf{E}_y = -2jE_o \sin\left(\frac{\pi x}{a}\right)\sin(\beta d) = 0$$

This can be satisfied if

$\beta d = l\pi \quad l = 1\ldots.\infty$

Here l is an integer. Substituting the value of β into this equation

$$\beta = \frac{l\pi}{d} = \sqrt{\omega_c^2 \mu\varepsilon - \left(\frac{\pi}{a}\right)}$$

This can only be satisfied for specific frequencies f_c (called resonant frequencies). Solving for f_c

$$(f_c)_{nol} = \frac{c}{2\pi}\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

Note that there is a specific frequency for each of the integers. Thus the resonant frequencies are labeled as f_{nol} . More general expression for TE_{nml} and TM_{nml} modes is

$$(f_c)_{nml} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \quad \text{(Hz)}$$

for TE_{nml} $\quad$ m or n $\neq 0$ $\quad$ $l > 0$

for TM_{nml} $\quad$ $m>0$, $n>0$, $l>0$

For Cylindrical cavity (a =radius, d= height)

$$f_c = \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}} \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \qquad \text{for } TE_{nml}$$

$$f_c = \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \qquad \text{for } TM_{nml}$$