

ECECS-611  
Classification of Transmission Lines  
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Classification of transmission lines are made with regards to the presence of the fields in the propagation direction. As a matter of convention, the direction of propagation is taken to be in the z-direction. All transmission lines can be rotated in space so that the direction of propagation coincides with the z-axis.

Typical transmission lines with this convention are shown in the Figure 1. Here, the rectangular waveguide, microstrip line, coaxial line and arbitrary two-wire transmission lines are shown.

A second important assumption is that the cross section of a given transmission line does not change in the z-direction, i.e., it maintains its cross section as one moves along the axial direction (direction of propagation).

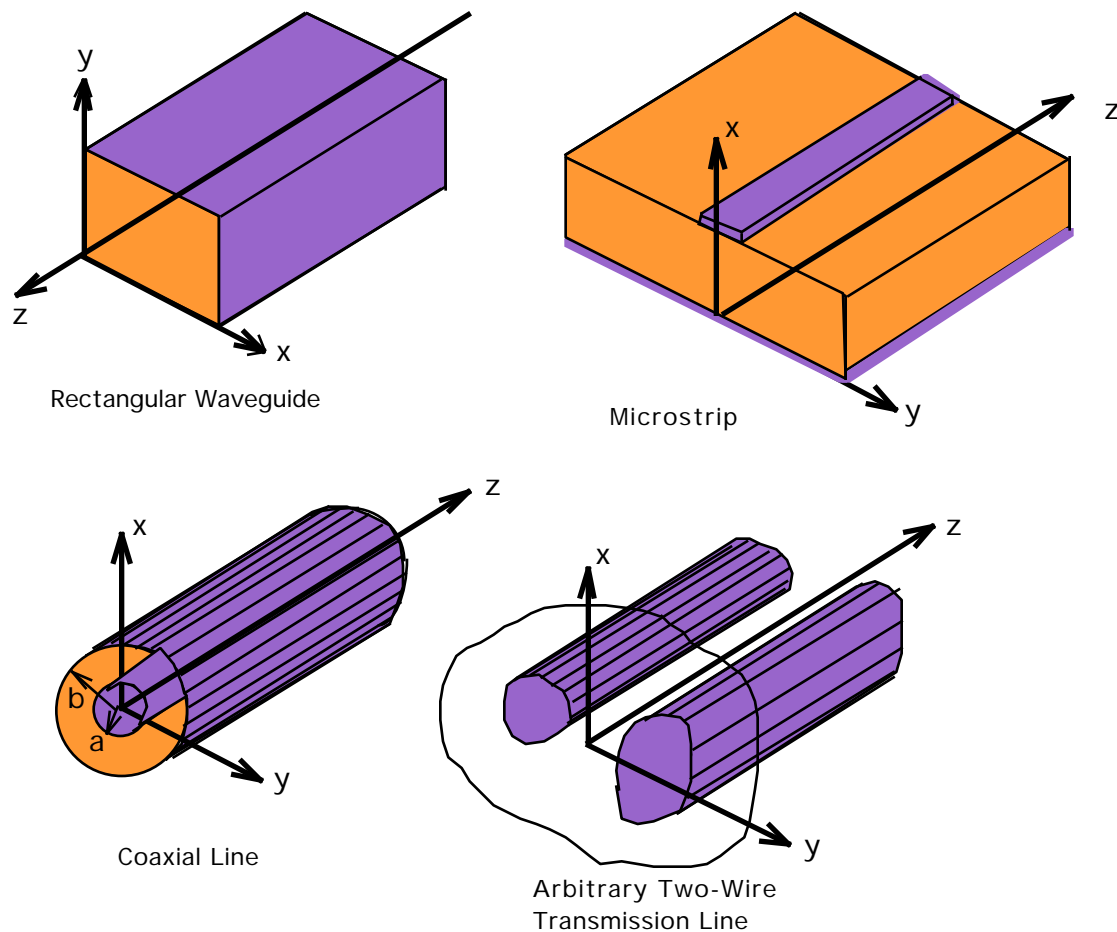


Figure 1. Various transmission lines.

The fields  $E(x,y,z)$  and  $H(x,y,z)$  are functions of the coordinates  $x$ ,  $y$  and  $z$ . In general, these fields have three components each, and each component  $E_x(x,y,z)$ ,  $E_y(x,y,z)$  and  $E_z(x,y,z)$  as well as  $H_x(x,y,z)$ ,  $H_y(x,y,z)$  and  $H_z(x,y,z)$  can all exist for a given transmission line.

We now look for field solutions with the following assumptions :

- 1) Initially, losses are neglected (Infinite conductivity approximation).
- 2) The fields propagate in the  $z$ -direction with a propagation constant  $\gamma$ .
- 3) The  $z$ -coordinate dependence of all the fields can be written as

$$e^{-j \gamma z}$$

4) The electric field has three components  $E_x(x,y,z)$ ,  $E_y(x,y,z)$  and  $E_z(x,y,z)$ . With the above  $z$  dependence, the total electric field can be written as

$$\begin{aligned} \mathbf{E}(x,y,z) &= \mathbf{E}_x(x,y,z) + \mathbf{E}_y(x,y,z) + \mathbf{E}_z(x,y,z) \\ &= \mathbf{e}_x(x,y) e^{-j \gamma z} + \mathbf{e}_y(x,y) e^{-j \gamma z} + \mathbf{e}_z(x,y) e^{-j \gamma z} \\ &= [\mathbf{e}_x(x,y) + \mathbf{e}_y(x,y)] e^{-j \gamma z} + \mathbf{e}_z(x,y) e^{-j \gamma z} \\ &= \mathbf{e}(x,y) e^{-j \gamma z} + \mathbf{e}_z(x,y) e^{-j \gamma z} \end{aligned}$$

Here,  $\mathbf{e}(x,y) = \mathbf{e}_x(x,y) + \mathbf{e}_y(x,y)$  is the transverse component (perpendicular to the direction of propagation) of the electric field and depend only on the transverse coordinates. The complete transverse component is actually  $E_t(x,y,z) = \mathbf{e}(x,y) e^{-j \gamma z}$ . Figure 2 shows these field components.

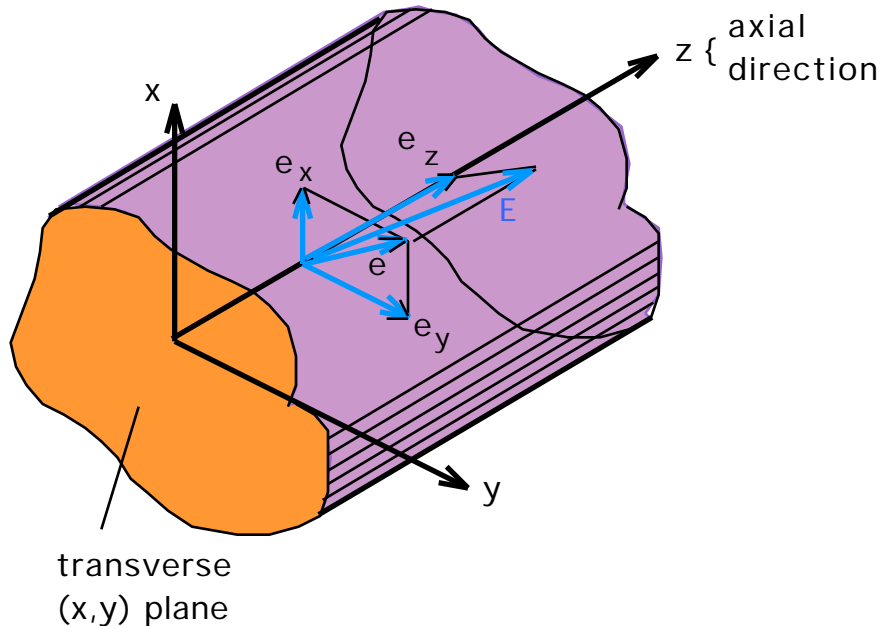


Figure 2.

Similarly the H field can also be written as

$$\begin{aligned} \mathbf{H}(x,y,z) &= \mathbf{H}_t(x,y,z) + \mathbf{H}_z(x,y,z) \\ &= \mathbf{h}(x,y) e^{-jz} + \mathbf{h}_z(x,y) e^{-jz} \end{aligned}$$

In order to write down Maxwell's equations in terms of the axial and transverse components, the Laplacian operator can also be written as

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - j^2 \mathbf{a}_z$$

$$= \nabla_t^2 - j^2 \mathbf{a}_z$$

Here,  $\nabla_t^2$  is the Laplacian operator related with the transverse coordinates and the z-component is obtained from the operation  $e^{(-jz)}/z$  since this is the only z-dependence that was assumed (transverse partial derivatives have no effect on the z-coordinate).

$$\nabla \cdot \mathbf{E} = (\nabla_t^2 - j^2 \mathbf{a}_z) \cdot \mathbf{E} = -j^2 \mu (\mathbf{h} + \mathbf{h}_z) e^{-jz}$$

$$\nabla_t \cdot \mathbf{E} - j^2 \mathbf{a}_z \cdot \mathbf{E} = -j^2 \mu \mathbf{h} - j^2 \mu \mathbf{h}_z$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \cdot \mathbf{E} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \cdot \mathbf{E} (\mathbf{e}_x + \mathbf{e}_y)$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \cdot \mathbf{E}_z$$

$$\mathbf{a}_z \cdot \mathbf{E} = \mathbf{a}_z \cdot \mathbf{E} (\mathbf{e}_x + \mathbf{e}_y)$$

The components are

$$\nabla_t \cdot \mathbf{E} = -j^2 \mu \mathbf{h}_z \quad (1)$$

$$\nabla_t \cdot \mathbf{E}_z - j^2 \mathbf{a}_z \cdot \mathbf{E} = -j^2 \mu \mathbf{h} \quad (2)$$

$$\nabla_t \cdot \mathbf{h} = j^2 \mathbf{e}_z \quad (3)$$

$$\nabla_t \cdot \mathbf{h}_z - j^2 \mathbf{a}_z \cdot \mathbf{h} = j^2 \mathbf{e} \quad (4)$$

$$\begin{aligned} \nabla \cdot \mathbf{B} &= \nabla \cdot \mu \mathbf{H} = (\nabla_t^2 - j^2 \mathbf{a}_z) \cdot (\mathbf{h} + \mathbf{h}_z) e^{-jz} \\ &= (\nabla_t \cdot \mathbf{h} - j^2 \mathbf{a}_z \cdot \mathbf{h}_z) e^{-jz} = 0 \end{aligned}$$

$$\nabla_t \cdot \mathbf{h} = j \quad h_z \quad (5)$$

$$\nabla_t \cdot \mathbf{e} = j \quad e_z \quad (6)$$

Through vector manipulations, Maxwell's equations are put in a form where the fields in the axial direction are related to the transverse components of the fields.

Classification of waves:

- 1) TEM (Transverse Electro-Magnetic) Waves  
both  $E_z=0$  and  $H_z=0$
- 2) TE (Transverse Electric) Waves  
 $E_z=0$ ,  $H_z=$  finite
- 3) TM (Transverse Magnetic) Waves  
 $H_z=0$ ,  $E_z=$  finite
- 4) Hybrid  
 $E_z$  and  $H_z$  both finite

## TEM Waves ( $h_z=0$ , $e_z=0$ )

Filed components for the TEM waves are calculated from

$$\mathbf{E}_t = \mathbf{e} e^{-jkz} = - \nabla_t e^{-jkz}$$

$$\mathbf{H}_t = Y_0 \mathbf{a}_z \times (\mathbf{e} e^{-jkz})$$

where the potential function satisfies

$$\nabla_t^2 \phi(x,y) = 0$$

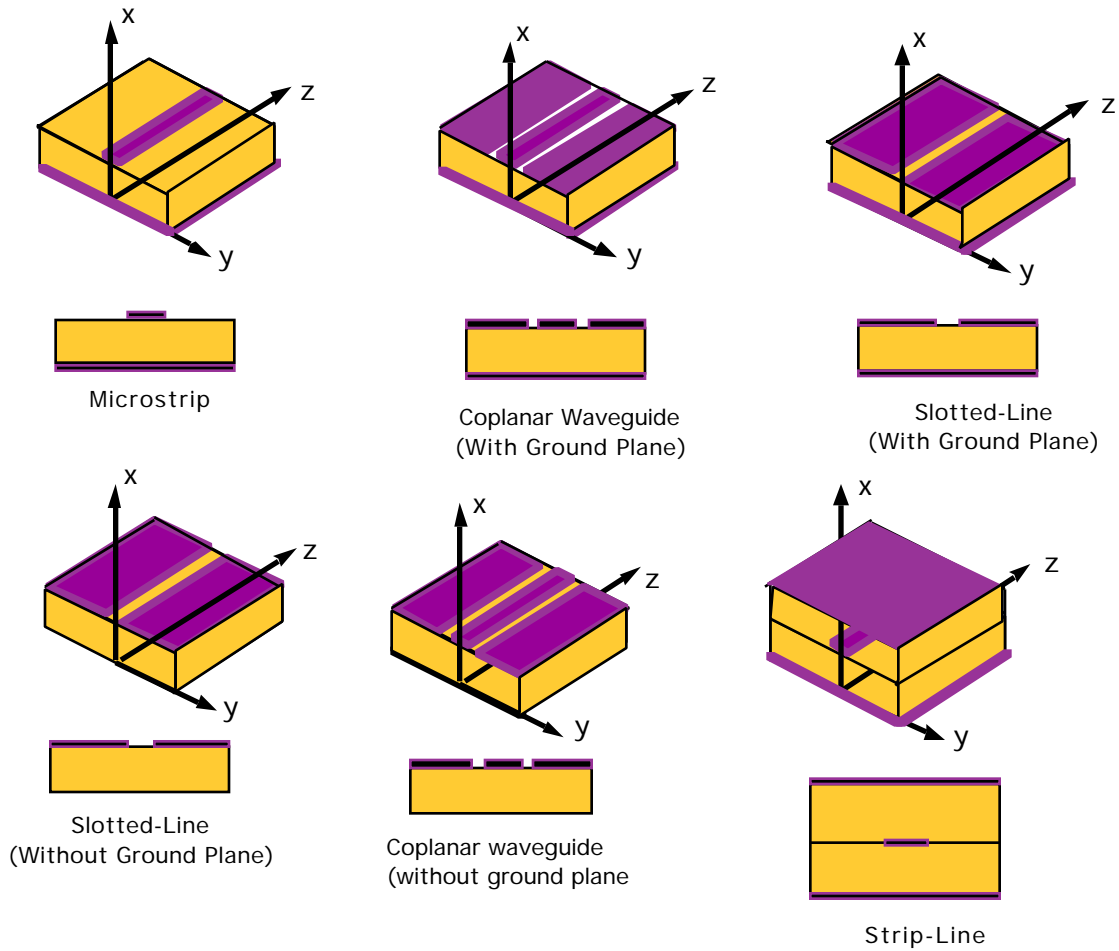
This is the same solution as the electrostatic field solution. Therefore, as a necessary (but not sufficient) condition, in order to transmit TEM waves, the structure should be able to support a static field distribution.

$$Y_0 = \frac{1}{Z_0} = \frac{1}{\sqrt{\mu/\epsilon}} = \sqrt{\epsilon/\mu} \quad (\text{wave admittance})$$

There is no lower limit on the frequency of transmission. Also the propagation constant for the TEM wave is the same as that of a plane wave. The  $\epsilon$  and  $\mu$  may be different than the material values and their effective values are substituted into the above equation.

The wave impedance for the TEM wave is the same as the plane wave. Eq. 11 gives the direction and magnitude of the  $\mathbf{h}$  field for the TEM wave.

### Common Planar Transmission Lines




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### TM Waves ( $h_z=0$ )

Helmholz equation is satisfied by both  $e_z$  and  $e_t$ .

$$\nabla_t^2 \mathbf{e}_t + (k^2 - \beta^2) \mathbf{e}_t = 0 \quad (7)$$

$$\nabla_t^2 e_z + (k^2 - \beta^2) e_z = 0 \quad (8)$$

We can define a new constant  $k_c^2 = k^2 - \beta^2$ . The solution of Equation 8 subject to the boundary conditions will determine the constant  $k_c$ . Once  $e_z$  is found from Eq.8,  $e_t$  and  $h$  can in principle be calculated.

Transverse components of  $e$  can be found from

$$\mathbf{e}_t = -\frac{j}{k_c^2} \nabla_t e_z \quad (9)$$

The h field is found from e field through

$$\mathbf{h} = \frac{1}{Z_e} (\mathbf{a}_z \times \mathbf{e}) = Y_e (\mathbf{a}_z \times \mathbf{e})$$

here

$$Y_e = \frac{1}{Z_e} = \frac{1}{\omega \mu} \sqrt{k^2 - k_c^2} \quad (\text{wave admittance for TM waves})$$

Therefore, transverse components of h can be found once the transverse components of e found from Eq.9.

From the definition of  $k_c$

$$k_c^2 = k^2 - \beta^2$$

Solving for the propagation constant

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{\omega^2 \mu \epsilon - k_c^2}$$

Depending on  $\omega$ , the propagation constant  $\beta$  can take on different values, i.e.,

$$\text{if } \omega > \frac{k_c}{\sqrt{\mu \epsilon}} \quad \beta \text{ is real} \quad e^{-j\beta z}$$

$$\text{if } \omega < \frac{k_c}{\sqrt{\mu \epsilon}} \quad \beta = -j\alpha \quad (\text{imaginary}) \quad e^{-\alpha z}$$

$$\omega_c = 2\pi f_c = \frac{k_c}{\sqrt{\mu \epsilon}} \quad \text{cut-off frequency}$$

Therefore, for TM waves, there is a minimum frequency (defined as the cut-off frequency) above which TM waves can propagate. Below the cut-off frequency determined by the  $k_c$ , no TM wave can propagate.

## TE Waves ( $e_z=0$ )

In this case,  $h_z$  behaves as the potential function

$$\nabla_t^2 h_z + (k^2 - \beta^2) h_z = 0$$

$$\nabla_t^2 h_z + k_c^2 h_z = 0$$

Transverse components of h can be found from

$$\mathbf{h} = -\frac{j}{k_c^2} \nabla_t h_z$$

Transverse components of  $\mathbf{e}$  can be found from

$$\mathbf{e} = -\frac{\mu}{k_c^2} (\mathbf{a}_z \times \nabla_t h_z) = -Z_m (\mathbf{a}_z \times \nabla_t h_z)$$

here

$$Z_m = \frac{\mu}{k_c} \quad (\text{wave impedance for TE waves})$$

$k_c$  and  $\omega_c$  are the same as for TM waves

$$k_c^2 = k^2 - \beta^2$$

or

$$= \sqrt{k^2 - k_c^2} = \sqrt{\omega^2 \mu - k_c^2}$$

Same propagation characteristics apply to the TE waves as in TM waves. There is a cut-off frequency below which no wave propagates. Note that the equations that determine the propagation constants are the same, but for the TM and TE waves,  $k_c$  may be different. Therefore, the cut-off frequencies may be different for these two waves.