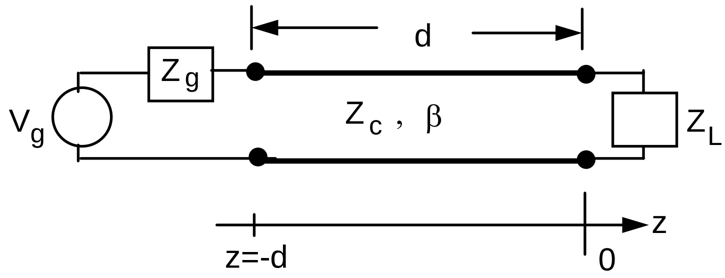
Example

Find the Power Delivered to a Load connected at the output of a lossless transmission line



Given the following parameters for a lossless transmission line:

Vg =15.0 (peak), Zc = 75 Ω, Yc = 1/Zc, ZL = (60 - j 40) Ω, Zg= 75 Ω, d = 0.7λ

Find the power delivered to the load using three different methods.

(a) Power delivered to the input of transmission line

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} \qquad \Gamma = -0.021 - j\,0.303$$

$$Z_{in} = Z_c \frac{Z_L + j\,Z_c \tan(\beta d)}{Z_c + J\,Z_L \tan(\beta d)} \qquad Zin = 48.187 + j27.32\ \Omega$$

Using the voltage division

$$P_{in} = \frac{1}{2} \mathrm{Re}\left[\left(\left| \frac{V_g}{Z_g + Z_{in}} \right| \right)^2 Z_{in} \right] \qquad Pin= 0.34\ W. \qquad (1)$$

(b) Using the formula for power delivered to the load

$$P_L = \frac{1}{2\,Z_c} \mid V^+ \mid^2 (1 - \mid \Gamma_L \mid^2) \qquad (2)$$

To find V^+ :

The current at the input of the transmission line is

$$I(z=-d) = \frac{V_g}{Z_g + Z_{in}} = \frac{1}{Z_c}\,[\,V^+ e^{j\beta d} - V^- e^{-j\beta d}\,]$$

$$= \frac{1}{Z_c} V^+ \left[\, e^{j\beta d} - \frac{V^-}{V^+} e^{-j\beta d} \right]$$

$$= \frac{1}{Z_c} V^+ [\, e^{j\beta d} - \Gamma_L e^{-j\beta d}]$$

Solving for V^+

$$V^+ = \frac{V_g}{(Z_g + Z_{in})\,(\,e^{-j\beta d} - \Gamma_L e^{j\beta d}\,)} Z_c \quad = -2.318 + j7.133 \ \ \text{V}$$

Substituting the values into Eq.2, we find PL= 0.34 W which is equal to Pin.

c) Using the Thevenin equivalent voltage. The open circuit voltage at the output of the line is

$$V_{th} = \frac{2\,V_g\,e^{-j\beta d}}{(1 + Y_c Z_g) + (\,1 - Y_c Z_g)\,e^{-j2\beta d}} \quad = -\,4.635 + j14.266$$

The Thevenin impedance is found by shorting the Vg and evaluating the impedance seen at the output terminals of the line with Zg as the load. This gives Zth=Zg, since the line characteristic impedance is also 75 Ω. The current flowing into the load is

$$I_L = \frac{V_{th}}{Z_{th} + Z_L} \quad = \ -\,0.06 + j\,0.088 \ \ \text{Amp}$$

Thus the power delivered to the load is

$$P_L = \frac{1}{2}\,\mathrm{Re}\left(\frac{|\,I_L\,|^2}{Z_L}\right) \ = \ 0.34\ \text{W} \qquad (3)$$

As can be seen that for a lossless line, the three different methods provide the same result.

Note: In this example Vg(peak)=15V used. If Vg(rms) is used the power calculated here will be multiplied by 2.