**EE-612-Microwave Communications Laboratory.**

# NOISE

The goal of information sciences is to transport information from one point to another. The information may be in many forms, and signals in the form of acoustic vibrations and electromagnetic waves constitute the main means of transporting the information. When two people talk, they transfer information from one person to the other through acoustic vibrations. When a radio station transmits sound or music, it is carried by electromagnetic signals from the transmitting antenna through space to the receiving antenna and through electronic processing equipment, to a loudspeaker which converts the signals back into acoustic vibrations so that we can hear the voice. The aim in information transmission is to transport the information with minimal interference from other sources.

When two people talk in a quiet room, they don't have to shout at each other to understand what each person is talking about. If there is a TV on in the room with a high volume, then the two people have to raise their voices to talk to each other. If you attend a crowded party, sometimes you have to raise your voice to an almost shouting level even in order to be heard by a person sitting next to you. The reason is that there are other people in the room talking to each other. Therefore, the speech of one person becomes noise to another person. Whenever some additional information interferes with the desired information and reaches the final destination together with the desired information, the unnecessary information is then considered noise for the party receiving that information.

Let us assume that with some magic we are able to remove all man-made noise and consider a voice modulated electromagnetic signal entering a receiver through an antenna. Even if the signal does not contain any noise by itself, it is possible that the individual components of the receiver may add noise to the signal while the signal is being amplified and processed by the receiving electronics. If the received signal amplitude is low, it is possible that the noise generated within the electronic components of the receiver may be comparable to or greater than the information signal amplitude. Eventually, although the signal amplitude is also amplified along the way, so is the noise and the signal may not be discernible among the larger noise amplitude.

The noise generated within the amplifying system that generally effects the overall signal performance is due to the noise contributions from the first few initial stages of an amplifier. By considering the noise properties of an electronic device, the noise contributions by the device and its low level signal amplifying properties can be determined. By optimizing the parameters of the device, the noise contribution can be minimized.

## A.1 NOISE POWER

Let us assume that a signal is transmitted across space and reaches a receiving antenna. The signal then enters the first stage of the receiving amplifier. We know that the receiving antenna has an equivalent input resistance and we can assume that this resistor is connected across the input terminals of the amplifying stage. As a matter of fact, we can assume that any other resistor may be connected across the amplifier input terminals.

A resistor is made up of a material which contains atoms of positive charges and negative electrons. Many of the electrons are bound to the atoms, but a few are free to move within the solid. Even if there are no external fields applied to the resistor, we know that at a given temperature, due to their internal thermal energy, these electrons are in thermal motion within the solid. Generally, the surrounding medium is at room temperature, but if power is dissipated within the resistor, the temperature of the resistor increases. This is reflected as a higher internal energy and more vibrant motion for the electrons. We also know that if charged particles are accelerated, they then radiate electromagnetic energy. If the energy of the radiated signal is $h\nu$, then the power radiated per unit time will be given by the probability that the electrons will have the energy $h\nu$ multiplied by the energy $h\nu$. The probability of having an energy $h\nu$ is given by the Bose-Einstein probability function (Eq. 3.9, Chapter 3). The power radiated is then

$$P(\nu) = \frac{h\nu \, d\nu}{e^{h\nu/kT} - 1} \qquad (A.1)$$

Electromagnetic signals that cover the present day communication channels range from low frequencies to millimeter wavelengths. If we take kT=0.026 eV (room temperature) we can practically assume that $h\nu/kT \ll 1$. Therefore, expanding the denominator of Eq. l, we obtain

$$P(\nu) = kT \, d\nu \qquad (A.2)$$

Therefore, the power radiated at the frequency $\nu$ within the frequency range $d\nu$ is independent of frequency and depends on temperature T only. At very high frequencies Eq. A.1 should be used.

Whenever the noise power over a very wide frequency range is constant, then the resulting noise is called the white noise. Note that any signal that has an amplitude below the power level given by equation A.2 will not be detected and lost within the background noise.

## A.2. JOHNSON NOISE

We can apply the concepts given above to the individual device components. Since one of the major dissipating components in an electronic device is a resistive element, we can represent the noise generated by the thermal electrons by an equivalent noise source.

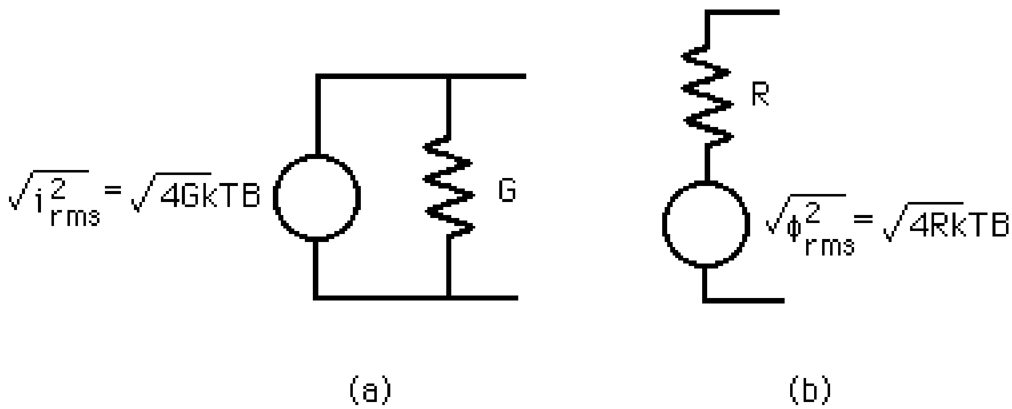


Figure A.1. Equivalent current and voltage noise sources in a resistor R=1/G.

Since noise is a result of random motion of the charged particles, the resulting noise signal has no d-c component. Therefore, we express the currents and voltages associated with noise by their root mean squares values. We can assume that the internally generated noise voltage is connected in series with a resistor R as shown in Fig. A.1.b Let the rms voltage be $\varphi_{rms}$. For maximum power transfer, the current fed into a second resistor of the same value R is $\varphi_{rms}/2R$ and the power dissipated in this resistor is $\varphi^2_{rms}/4R$. If this is equated to the thermal power given by Eq. 11.2, then

$$\tilde{\varphi}^2_{rms} = 4RkTB \qquad (A.3)$$

where we have used B instead of $d\nu$ where B is the bandwidth.

Similarly, using Thevenin's equivalent theorem, a current noise source given by

$$\tilde{i}^2_{rms} = 4GkTB \qquad (A.4)$$

can be used in parallel with a conductance G as shown in Fig.A.1.a. The noise voltage or the current represented by Eq.A.3 and A.4 are known as the Johnson noise.

A.3. SHOT NOISE

The random generation and directed motion of charge carriers leads to shot noise. Here the random fluctuation of moving charge carriers constitute the shot noise and as expected depends on the amount of charge flowing, i.e., to current. It is given by an equivalent noise current

$$\tilde{i}^2_s(\nu) = 2e\bar{I}B \qquad (A.5)$$

Here I is the average current produced by the moving charges.

A.4. 1/f NOISE.

There is another source of noise which becomes important at low frequencies. This is referred to as 1/f noise. The origin of this noise can be attributed to random fluctuations of gain, capacitance, voltage source, etc. Although one can question the validity of these arguments, by modeling these drifts properly, one can show an increase in noise as the frequency decreases. Experimental measurements also prove the existence of 1/f noise.

## A.5 NOISE FIGURE

Whenever signals are applied to amplifiers, the noise generated by an amplifier by far exceeds the thermal noise discussed above. The electrons within the device which determine the amplifying characteristics of the device contribute to the internal noise to the device. Depending on the device configuration and mode of operation, the noise sources and their effective contributions vary from one device to the other. The noise contribution by an amplifier is identified by a noise figure F defined by

$$F = \frac{\text{signal to noise ratio at input}}{\text{signal to noise ratio at output}} \qquad \text{(A.6)}$$

In order to simplify the analysis, the noise generated by an amplifier is assumed to be applied to an idealized amplifier with no internal noise and the resulting noise figure is calculated.

In Fig. 11.2, an input signal S and an input noise $N_o$ are connected across the terminals of an amplifier. Also the noise $N_1$ generated by the amplifier is also connected as a source at the input of the amplifier. The gain of the amplifier is now $G_1$ and the amplified noise is $(N_o+N_1)G_1$. The noise figure of the amplifier is then given by

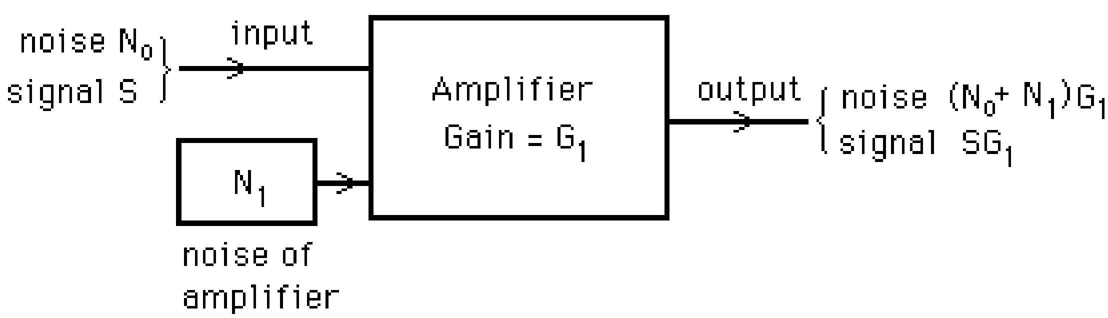


Figure A.2. Ideal representation of an amplifier with noise $N_1$.

$$F_1 = \frac{S/N_o}{SG_1/\{(N_o+N_1)G_1\}}$$

$$F_1 = 1 + \frac{N_1}{N_o} \qquad \text{(A.7)}$$

If we measure every noise source with respect to kTB, then we can write $N_i = N'_i kTB$ and Eq. 6 becomes

$$F_1 = 1 + N'_1 \qquad (A.8)$$

Therefore the contribution to the overall noise by the amplifier is given by $N'_1$.

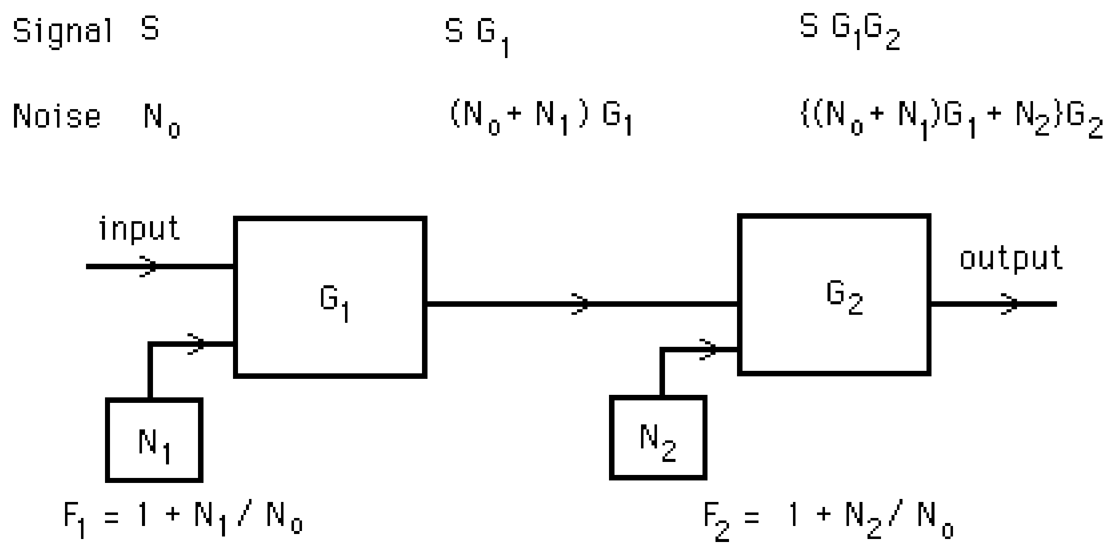


Figure A.3. Overall noise figure of a two stage cascade amplifier.

Whenever two amplifiers are connected in cascade, the overall noise figure can be calculated by referring to Fig. A.3. Using the indicated signal and noise at the output, the overall noise figure of a two stage amplifier can be written as

$$F_{overall} = \frac{S/N_o}{SG_1G_2 / \{(N_o + N_1)G_1 + N_2\}G_2}$$

$$F_{overall} = \frac{N_o + N_1 + (N_2/G_1)}{N_o}$$

$$F_{overall} = 1 + N'_1 + \frac{N'_2}{G_1} \qquad (A.9)$$

Using the noise figure for each stage

$$F_{overall} = F_1 + \frac{F_2 - 1}{G_1} \qquad (A.10)$$

Therefore the overall noise figure is determined by the gain of the first amplifier stage in addition to noise figures of both stages. But if the gain of the first stage is high, then the contribution of the second stage on the overall noise figure of the system can be neglected.

If we let $T_o = N'_o T = 1$ in $N_o = N'_o kTB$ to be our reference temperature, then the temperature added by the two stages is

$$T_1 + \frac{T_2}{G_1} \qquad (A.11)$$

and is known as excess temperature added by the amplifiers. Sometimes the reference temperature is taken to be 190 K or 300 K and excess noise temperature is measured relative to this. If the source is connected to a source $T_s$ other than the reference temperature, then the overall noise temperature of the receiver is

$$T_s + T_1 + \frac{T_2}{G} \qquad (A.12)$$

Noise figure of various electronic devices and their origins are covered many textbooks. A detailed description of these are beyond the scope of this book. For starters, interested readers are referred to the references at the end of this Appendix.

REFERENCES:


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A.3. S.D. Senturia & B.D.Wedlock, Electronic Circuits and Applications , John Wiley & Sons, N.Y. (1975).

A.2. J.B.Johnson, "Thermal Agitation of Electricity in Conductors", Phys, Rev, Vol.32, 97-109(1928).

A.3. H. Nyquist, "Thermal Agitation of Electrical Charge in Conductors", Phys. Rev. Vol.32, 110-113(1928).


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Noise Figure

The noise figure of a microwave transistor can be written as

$$F = F_o \left\{ 1 + \left( \frac{f}{f_c} \right)^2 \right\} \qquad (12.5)$$

Here $F_o$ is the low frequency noise figure of the transistor. It is constant up to a critical frequency $f_c$ and increases 6 dB per octave above $f_c$. $F_o$ depends largely on $h_{fe}$ of the transistor and decreases with increasing $h_{fe}$. Typical noise figure variation of a transistor as a function of frequency is shown in Fig.12.2.

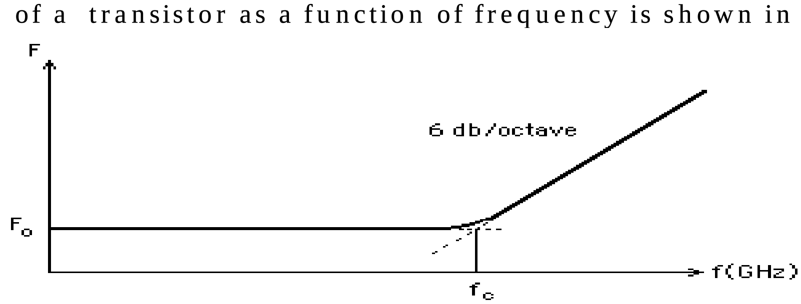

Figure A.4. Noise figure of a microwave transistor as a function of frequency.

Noise figure of a two port network is given by

$$F = F_{min} + \frac{R_N}{G_s} \, | Y_s - Y_{opt} |^2$$

here $F_{min}$ is the minimum noise figure obtained when the input impedance has a value of $Y_{opt}$. $Y_s$ is the input admittance connected to the input of the two port, $G_s$ = Re ($Y_s$) and $r_N$ is a parameter.

Let

$$Y_s = \frac{1}{Z_o} \left[ \frac{1 - \Gamma_s}{1 + \Gamma_s} \right]$$

$$Y_{opt} = \frac{1}{Z_o} \left[ \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}} \right]$$

then

$$Y_s - Y_{opt} = \frac{2 ( \Gamma_s - \Gamma_{opt} )}{( 1 + \Gamma_s ) ( 1 + \Gamma_{opt} )}$$

also

{ EMBED Word.Picture.6 }

Substituting these back into the original equation

$$F = F_{min} + \frac{4 R_N}{Z_o} \, \frac{| \Gamma_s - \Gamma_{opt} |^2}{( 1 - | \Gamma_s |^2 ) ( | 1 + \Gamma_{opt} |^2 )} \qquad \text{(N1)}$$

If the source is matched to $Z_o$, the noise figure becomes

$$F_{\Gamma_s = 0} = F_{min} + 4 r_N \frac{| \Gamma_{opt} |^2}{| 1 + \Gamma_{opt} |^2} \qquad \text{(N2)}$$

here $r_N = R_N / Z_o$. Solving for $r_N$

$$r_N = \frac{( F_{\Gamma_s = 0} - F_{min} )}{4} \, \frac{| 1 + \Gamma_{opt} |^2}{| \Gamma_{opt} |^2}$$

Arranging (N2) and defining $N_i$ as

$$N_i = \frac{| \Gamma_s - \Gamma_{opt} |^2}{1 - | \Gamma_s |^2} = \frac{F - F_{min}}{4 r_N} \, | 1 + \Gamma_{opt} |^2$$

constant noise figure circles can be calculated. The center and radius are given by

$$C_F = \frac{\Gamma_{opt}}{N_i + 1}$$

$$R_F = \frac{\sqrt{N_i ( N_i + 1 - | \Gamma_{opt} |^2 )}}{N_i + 1}$$